Make sure you know:

$$\mathcal{L}[y(x)] = Y(s)$$

$$\mathcal{L}[y'] = -y(0) + sY$$

$$\mathcal{L}[y''] = -y'(0) - s\,y(0) + s^2 Y$$

Interpreting Matrix solutions (eqs. = # variables)

$$\left[\begin{array}{cccccc|c} 1 & \cdots & & & & & \cdots \\ 0 & 1 & \cdots & & & & \cdots \\ 0 & 0 & \cdots & & & a & b \end{array}\right]$$

3 cases!

$a = b = 0$: ∞-ly many solutions

$a = 0,\ b \neq 0$: no solution

$a \neq 0$: exactly one solution

Q8 #18

$$\begin{aligned} -x + y - z &= -4 \\ 2x - 2y + 2z &= 8 \\ x - y + z &= 4 \end{aligned} \longrightarrow \left[\begin{array}{ccc|c} -1 & 1 & -1 & -4 \\ 2 & -2 & 2 & 8 \\ 1 & -1 & 1 & 4 \end{array}\right]$$

$$\xrightarrow{-R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 2 & -2 & 2 & 8 \\ 1 & -1 & 1 & 4 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \begin{array}{c} \begin{array}{cccc} & y & z & \end{array} \\ \left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$x - y + z = 4$$

$$\left[\begin{aligned} y &= s \\ z &= t \end{aligned}\right.$$

$$x - s + t = 4 \longrightarrow x = 4 + s - t$$

$$\boxed{\begin{aligned} x &= 4 + s - t \\ y &= s \\ z &= t \end{aligned}}$$

Q9 #10

$$y'' - 2y' + 10y = 3 + 2e^{-2x}$$

$$y(0) = 4 \qquad y'(0) = 4$$

$$-y'(0) + 2y(0)s + s^2 Y + 2y(0) - 2sY + 10Y = \frac{3}{s} + \frac{2}{s+2}$$

$$-4 + 8s + s^2 Y + 8 - 2sY + 10Y = \frac{5s+6}{s(s+2)}$$

$$(s^2 - 2s + 10)Y = \frac{5s+6}{s(s+2)} - 4$$

$$(s^2 - 2s + 10)Y = \frac{-4s^2 - 3s + 6}{s(s+2)}$$

$$Y = \frac{-4s^2 - 3s + 6}{s(s+2)(s^2 - 2s + 10)}$$

$$\frac{-4s^2-3s+6}{s(s+2)(s^2-2s+10)} = \frac{A}{s} + \frac{B}{s+2} + \frac{Cs+D}{s^2-2s+10}$$

$$-4s^2-3s+6 = A(s+2)(s^2-2s+10) + Bs(s^2-2s+10) + \underbrace{Cs^2(s+2) + Ds(s+2)}_{(Cs+D)s(s+2)}$$

$s=0$: $6 = A(2)(10)$

$\frac{3}{10} = A$

$s=-2$:

$$\boxed{\text{etc.}}$$

Q8 # 12

$$\begin{aligned} 2x - 8y &= -4 \\ x - 4y &= 2 \\ -\tfrac{1}{2}x + 2y &= -1 \end{aligned} \longrightarrow \left[\begin{array}{cc|c} 2 & -8 & -4 \\ 1 & -4 & 2 \\ -\tfrac{1}{2} & 2 & -1 \end{array}\right]$$

$$\xrightarrow[2R_3]{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|c} 1 & -4 & 2 \\ 2 & -8 & -4 \\ -1 & 4 & -2 \end{array}\right] \xrightarrow[R_3 + R_1]{R_2 - 2R_1} \left[\begin{array}{cc|c} 1 & -4 & 2 \\ 0 & 0 & -8 \\ 0 & 0 & 0 \end{array}\right]$$

$0 = -8$

$\boxed{\text{no solution}}$

Q8 #19

$$\begin{aligned} 4x - 5y - 2z &= -2 \\ -5x + 3y + (2+k)z &= -4 \\ -2x + 2y + z &= 0 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 4 & -5 & -2 & -2 \\ -5 & 3 & 2+k & -4 \\ -2 & 2 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_1/4} \left[\begin{array}{ccc|c} 1 & -5/4 & -1/2 & -1/2 \\ -5 & 3 & 2+k & -4 \\ -2 & 2 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{\substack{R_2+5R_1 \\ R_3+2R_1}} \left[\begin{array}{cccc} 1 & -5/4 & -1/2 & -1/2 \\ 0 & -13/4 & k-1/2 & -3/2 \\ 0 & -1/2 & 0 & -1 \end{array}\right]$$

$$\xrightarrow[R_3 \cdot 2]{R_2 \cdot 4} \left[\begin{array}{ccc|c} 1 & -5/4 & -1/2 & -1/2 \\ 0 & -13 & 4k-2 & -6 \\ 0 & -1 & 0 & -2 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -5/4 & -1/2 & -1/2 \\ 0 & -1 & 0 & -2 \\ 0 & -13 & 4k-2 & -6 \end{array}\right]$$

$$R_2(-1) \left[\begin{array}{ccc|c} 1 & -5/4 & -1/2 & -1/2 \\ 0 & 1 & 0 & 2 \\ 0 & -13 & 4k-2 & -6 \end{array}\right]$$

$$\xrightarrow{R_3+13R_2} \left[\begin{array}{ccc|c} 1 & -5/4 & -1/2 & -1/2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 4k-2 & 20 \end{array}\right]$$

$$(4k-2)z = 20$$

$4k-2 = \underbrace{20 = 0}_{\text{not possible}}$ → ∞-ly many solutions

$\underbrace{4k-2=0}_{\to\, k=\frac{1}{2}}$, $\underbrace{20 \neq 0}_{\to\, \text{true}}$ → no solution

$4k-2 \neq 0$ → one solution

→ $k \neq \frac{1}{2}$

Pop 12: AAAAA

Q8# 16

$$3x + y - z = 0$$
$$-2x - 4y + 2z = 6$$
$$x + 2y - z = -3$$

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & 0 \\ -2 & -4 & 2 & 6 \\ 1 & 2 & -1 & -3 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ -2 & -4 & 2 & 6 \\ 3 & 1 & -1 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 - 3R_1]{R_2 + 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & -5 & 2 & 9 \end{array}\right]$$

$x = s$ or $y = s$ or $z = s$

$x = s \rightarrow z = s + 2y + 3$

$$2z = 5y + 9$$

$$2z = 2s + 4y + 6$$

$$2s + 4y + 6 = 5y + 9$$
$$2s - 3 = y$$

$$z = s + 2(2s - 3) + 3 = 5s - 3$$

$$x = s$$
$$y = 2s - 3$$
$$z = 5s - 3$$

Q9 #20 non-trivial (not all zero)

$$\begin{aligned} -2x+3y-z&=0 \\ -8x+ay-4z&=0 \\ 2x+3y-3z&=0 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} -2 & 3 & -1 & 0 \\ -8 & a & -4 & 0 \\ 2 & 3 & -3 & 0 \end{array}\right]$$

$$\xrightarrow{R_1(-\frac{1}{2})} \left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ -8 & a & -4 & 0 \\ 2 & 3 & -3 & 0 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2+8R_1} \left[\begin{array}{ccc|c} 1 & -3/2 & \frac{1}{2} & 0 \\ 0 & a-12 & 0 & 0 \\ 0 & 6 & -4 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -3/2 & \frac{1}{2} & 0 \\ 0 & 6 & -4 & 0 \\ 0 & a-12 & 0 & 0 \end{array}\right] \xrightarrow{R_2/6} \left[\begin{array}{ccc|c} 1 & -3/2 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{2}{3} & 0 \\ 0 & a-12 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_3-(a-12)R_2} \left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3}(a-12) & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3}(a-12) & 0 \end{array}\right]$$

We know $(0,0,0)$ is a solution

if $\frac{2}{3}(a-12) \neq 0 \rightarrow$ 1 solution $\rightarrow (0,0,0)$

<u>not</u> what we want

$\frac{2}{3}(a-12) = 0 \rightarrow \infty$-ly many solutions

$a - 12 = 0$

$\boxed{a = 12}$

$$\begin{vmatrix} -2 & 3 & -1 \\ -8 & a & -4 \\ 2 & 3 & -3 \end{vmatrix} = 6a - 24 + 24 - 24 - 72 + 2a$$

$$= 8a - 96$$

$8a - 96 \neq 0$: one exact solution $\rightarrow (0,0,0)$

$8a - 96 = 0$: ~~no solution~~ or $\boxed{\infty\text{-ly many}}$

$(0,0,0)$

$8a = 96$

$a = 12$