Math 132 Final Examination Fall 2000

This exam should have 18 multiple choice questions and 5 True-False questions. There are **no** hand graded problems. Please check to see that your exam is complete. If you do not have a **PENCIL** to mark your card, please ask to borrow one from your proctor.

Write your social security number on the nine blank lines at the top of your answer card, using one blank for each digit. **Shade in the corresponding boxes below.** Print your name at the top of your card. Leave the boxes for version type blank.

As you work the exam, lightly shade in the correct answers on your answer card. At the end of the exam, when you are certain of all your choices, darken all your answer boxes. If your card becomes damaged please ask your proctor for a new one.

1. $\int e^x (1 + e^x)^4 \, dx =$

A) 1

B) $e^x + c$

C) c

D) $ln\, x + c$

E) $ln\, x$

F) $\frac{(1+e^x)^2}{2} + c$

G) e^x

H) $\frac{1}{5}(1 + e^x)^5 + c$

I) $ln\,|x + 1| + c$

J) $ln\,(x + 1) + c$

2. $\int x^2 \sin x \, dx =$

A) $x^2 \sin x \cos x + c$

B) $2 \cos x - 2x^2 \sin x + c$

C) $x \cos x + (1 - x) \sin x + c$

D) $(x + 1) \cos x - \sin x + c$

E) $x^2 \cos x - x \sin x + c$

F) $(2 - x^2) \cos x + 2x \sin x + c$

G) $\frac{x^3}{3} \sin x + c$

H) $- x^2 \cos x + c$

I) $2x \cos x + x^2 \sin x + c$

J) $2x \sin x + c$

3. $\int \frac{dx}{x(x-1)} =$

A) $\frac{1}{2x-1} + c$

B) $ln\left|\frac{x-1}{x}\right| + c$

C) $csc^{-1}x + c$

D) $10\,ln\,|x| + 5\,ln\,|x-1| + c$

E) $e^x + c$

F) $ln\,(sec\,x + tan\,x) + c$

G) $\frac{1}{x-1} - \frac{1}{x} + c$

H) $ln\,|x(x-1)| + c$

I) $ln\,|x| - ln\,|x-1| + c$

J) $\frac{1}{x^2} + \frac{1}{(x-1)^2} + c$

4. Find the derivative of $g(x) = \int_0^{x^2} \frac{1}{1+t}\, dt$.

A) $tan^{-1}(x) + c$

B) $sin^{-1}(x) + c$

C) 0

D) 1

E) $ln\,|1 + x|$

F) $\frac{2x}{1+x^2}$

G) $\frac{2}{1+x^2}$

H) $\frac{2x}{1+x}$

I) $\frac{1}{1+x^2}$

J) $g\prime(x)$ doesn't exist.

5. Use Simpson's rule with $n = 4$ (i.e., 4 partitions) to estimate the area under the curve $y = e^x$ from $x = 1$ to $x = 3$. (Do not evaluate it using the Evaluation Theorem.)

A) 17.279

B) 17.311

C) 17.367

D) 17.373

E) 17.394

F) 17.415

G) 17.453

H) 17.482

I) 17.509

J) 17.525

6. A cylindrical tank of base radius 3 ft and height of 6 ft is filled with oil. The oil weighs 100 lb/ft^3. Find the work required to pump all the oil to the top of the tank. (Answer in ft-lb.)

A) $10{,}120\pi$

B) $43{,}270\pi$

C) $16{,}200\pi$

D) $18{,}000\pi$

E) $8{,}270\pi$

F) $12{,}410\pi$

G) $28{,}280\pi$

H) $20{,}210\pi$

I) $14{,}400\pi$

J) $6{,}800\pi$

7. A specific light bulb has an average lifetime of 1000 hours. The probability of failure is modeled by an exponential density function. Find the probability that one of these bulbs lasts more than 1000 hours.

A) 71.2%

B) 70.5%

C) 64.1%

D) 59.7%

E) 50%

F) 43.4%

G) 41.2%

H) 36.8%

I) 22.3%

J) 18.1%

8. Use Euler's method with step size 0.2 to estimate $y(0.4)$, where $y(x)$ is the solution of the initial value problem $y\prime = 2xy^2$, $y(0) = 1$.

A) 0.91

B) 1.21

C) 2.27

D) 1.08

E) 1.55

F) 0.76

G) 2.42

H) 0.21

I) 1.36

J) 1.80

9. Let $u(t)$ be the solution to

$$\frac{du}{dt} = e^{u+t}$$

with $u(0) = 0$. Find $u(\frac{1}{2})$. (Hint: Note that this equation is separable.)

A) 0

B) 1

C) π

D) $\sqrt{e}$

E) $-\, ln(2 - \sqrt{e}\,)$

F) $2^{2\sqrt{e}}$

G) $\sqrt{2e}$

H) 3

I) 4

J) $\frac{12-e}{5}$

10. A nuclear power plant produces water which contains a dangerous radioactive isotope in concentrations 12 times too great for it to be released in the local river. The isotope has a half life of 200 years. For how many years must the water be stored before it can be released.

A) 692

B) 709

C) 703

D) 717

E) 733

F) 757

G) 762

H) 815

I) 902

J) 902

11. Determine the convergence or divergence of these sequences and series:

$$\left\{\frac{1}{n}\right\}_{n=0}^{\infty}, \quad \left\{\frac{(-1)^{n-1}}{n}\right\}_{n=0}^{\infty}, \quad \sum_{n=1}^{\infty}\frac{1}{n}, \quad \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}$$

Respectively,

A) D D D C

B) D C D C

C) C D C D

D) C C D D

E) C C D C

F) C D D D

G) C D D C

H) D D C C

I) C C C C

J) D C D D

12. $7 - \frac{7}{6} + \frac{7}{36} - \frac{7}{216} + \frac{7}{1296} - \cdots =$

A) 5.72

B) 5.86

C) 5.93

D) 6.00

E) 6.05

F) 6.13

G) 6.19

H) 7.34

I) 8.24

J) The series diverges.

13. Estimate $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$ to within 0.01.

A) 2.17

B) 0.76

C) 0.90

D) 1.03

E) 1.37

F) 0.85

G) 0.58

H) 2.71

I) 0.82

J) 1.14

14. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-4)^n}{n\,5^n}$ is

A) $(-4, 4)$

B) $[-5, 5)$

C) $(0, 4]$

D) $(-1, 9)$

E) $[-4, 4)$

F) $(-5, 5)$

G) $[-1, 9)$

H) $(-5, 5]$

I) $(-4, 4]$

J) $(-1, 9]$

15. Which of these is a power series for $\frac{tan^{-1}x}{x}$ at $a = 0$?

A) $1 - x^2 + x^4 - x^6 + x^8 - \cdots$

B) $1 - \frac{x^2}{3} + \frac{x^4}{5} - \frac{x^6}{7} + \frac{x^8}{9} - \cdots$

C) $1 - 2x^2 + 4x^4 - 6x^6 + 8x^8 - \cdots$

D) $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots$

E) $x - \frac{x^3}{4} + \frac{x^5}{6} - \frac{x^7}{8} + \frac{x^9}{10} - \cdots$

F) $x - 4x^3 + 6x^5 - 8x^7 + 10x^9 - \cdots$

G) $1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \frac{x^4}{5} - \cdots$

H) $1 - 2x + 3x^2 - 4x^3 + 5x^4 - \cdots$

I) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots$

J) $x - x^2 + x^3 - x^4 + x^5 - \cdots$

16. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{2^{2n+1} \cdot (2n+1)!} =$

A) 1

B) $\frac{1}{2}$

C) $-\frac{1}{2}$

D) -1

E) 0

F) $\frac{\sqrt{2}}{2}$

G) $-\frac{\sqrt{2}}{2}$

H) $\frac{3}{2}$

I) $-\frac{3}{2}$

J) π

17. What is the Taylor coefficient c_3 of the Taylor series of $f(x) = \sqrt{x}$ at $a = 1$?

A) $\frac{1}{6}$

B) $\frac{3}{4}$

C) $-\frac{1}{5}$

D) $\frac{1}{3\sqrt{2}}$

E) 0

F) $-\frac{1}{2}$

G) $\frac{1}{16}$

H) $-\frac{1}{24}$

I) $\frac{1}{2\sqrt{2}}$

J) $\frac{1}{4}$

18. $\int_0^1 cos\,(x^2)\,dx\ =$

A) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \cdots$

B) $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!} - \cdots$

C) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$

D) $1 - \frac{1}{2\cdot 3!} + \frac{1}{4\cdot 5!} - \frac{1}{8\cdot 7!} + \frac{1}{16\cdot 9!} - \cdots$

E) $1 - \frac{1}{5\cdot 2!} + \frac{1}{9\cdot 4!} - \frac{1}{13\cdot 6!} + \frac{1}{17\cdot 8!} - \cdots$

F) $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} - \cdots$

G) $1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots$

H) $1 - \frac{1}{(3!)^2} + \frac{1}{(5!)^2} - \frac{1}{(7!)^2} + \frac{1}{9!)^2} - \cdots$

I) $1 - \frac{2}{2!} + \frac{4}{4!} - \frac{8}{6!} + \frac{16}{8!} - \cdots$

J) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$

19. True or False: Let f and g be functions that are continuously differentiable on the real line. Suppose for x outside the open interval (a, b), one has that $f(x) = g(x) = 0$. Then

$$\int_a^b f(x)\ g\prime(x)\,dx = -\int_a^b f'(x)\,g(x)\,dx.$$

A) True

B) False

20. True or False: If $f(x)$ is continuous on $[1, 4]$, and $\int_1^4 f(x)\,dx = -48$, there must exist a $c \epsilon [1, 4]$ such that $f(c) = -12$.

A) True

B) False

21. True or False: $y = -2x + \frac{2}{x}$ is the solution of the differential equation $x^2y\prime = xy - 4$ with initial condition $y(2) = -3$.

A) True

B) False

22. True or False: If $\lim_{n\to\infty} a_n = 0$, then

$$\sum_{n=1}^{\infty} a_n,$$

converges.

A) True

B) False

23. True or False: The series $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ converges absolutely.

A) True

B) False