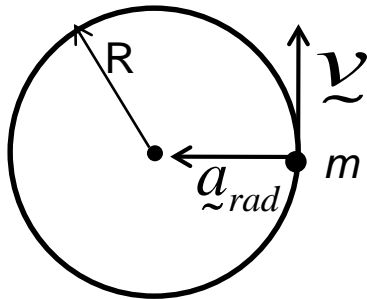


Lecture 7: Applications of Newton's Laws

Chapter 5, Sections 5.4 - 5.5 of the Text

Dynamics of Circular Motion



Since there is a \underline{a}_{rad} , there must be a radial force \underline{F}_{rad} pointing in the same direction. Thus, we write for the 2nd Newton law

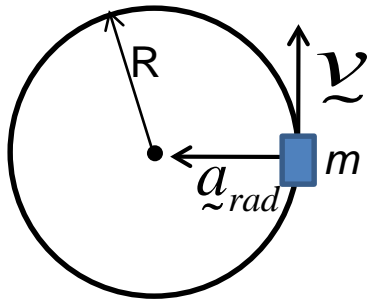
$$\underline{F}_{rad} = m\underline{a}_{rad} = \frac{mv^2}{R} \quad \text{direction to the center} \quad (5.24)$$

One is often interested in how long it takes to go around the circle in uniform circular motion, i.e., what is the period T ,

$$T = \frac{2\pi R}{v} \quad (5.25)$$

In terms of the period the radial acceleration can be expressed as

$$a_{rad} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2} \quad (5.26)$$



Rounding a flat curve

How can a car negotiate a flat curve? What keeps the car on the curve?

It is the centripetal force due to **static friction** force f_s^{\max} .

$$f_s^{\max} = \mu_s N \equiv \text{centripetal force} = \frac{mv^2}{R} \quad (5.27)$$

Since in the vertical direction the weight is opposed by the normal force N ,

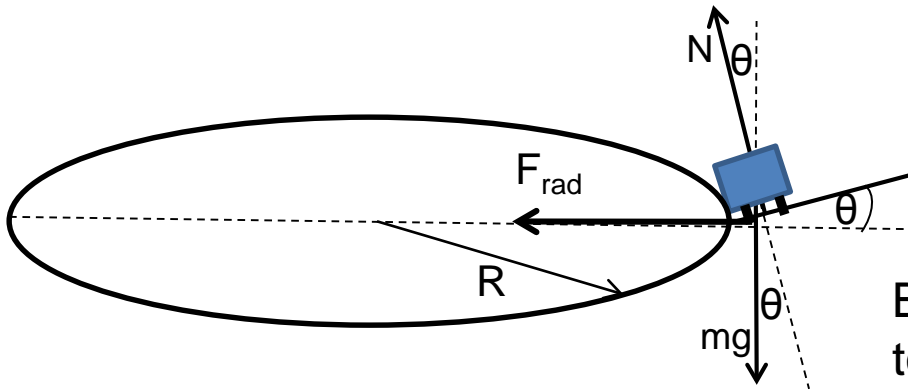
$$N - mg = 0 \quad (5.28)$$

Substituting from 5.28 into 5.27, we finally get for the maximum speed with which we can negotiate the curve without slipping

$$v_{\max} = \sqrt{\mu_s Rg} \quad (5.29)$$

What if the curve is not flat but is banked?

Rounding a banked curve



$$F_{rad} = N \sin \theta = \frac{mv^2}{R} \quad (5.30)$$

But, since there is no motion perpendicular to the banked curve,

$$N \cos \theta - mg = 0 \quad (5.31)$$

Combining Eqs. 5.30 and 5.31, we obtain

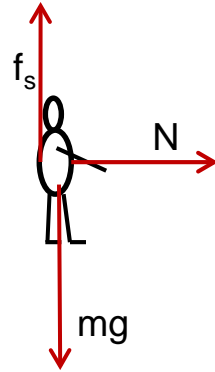
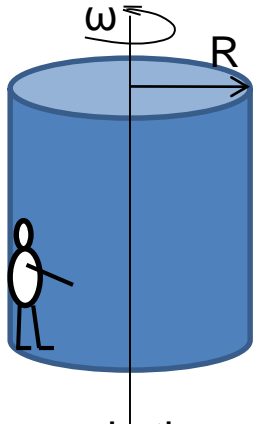
$$v_{\max} = \sqrt{Rg \tan \theta} \quad (5.32)$$

For maximum speed on a banked round neglecting any friction.

The car navigates on the banked curve purely due to the horizontal component of the normal force the banked road exerts on the car.

Example problems

P. 5.115: On the ride “Spindletop”, people stand against the inner wall of a hollow vertical cylinder with radius 2.5 m. The cylinder starts to rotate about its vertical axis and when it reaches a constant rotation rate of 0.60 rev/s, the floor on which people were standing drops some 0.5 m. The people remain pinned against the wall. (a) Draw a free-body diagram for a person on this ride after the floor has dropped. (b) What minimum coefficient of static friction is required if the person on the ride is not to slide downward.



The person moves on a horizontal circle due to the horizontal force N (normal force) pointing to the center of the circle. Hence, his radial acceleration is

$$N = ma_{rad} = \frac{mv^2}{R}$$

Forces in the vertical direction satisfy $f_s^{\max} - mg = 0$ (since the person is pinned to the wall).

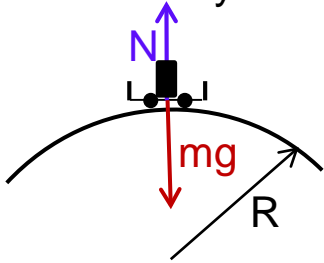
Maximum static friction is $f_s^{\max} = \mu_s N$. Thus, substituting for f_s^{\max} and N we obtain

$$\mu_s m \frac{v^2}{R} = mg \quad \text{from which the friction coefficient is } \mu_s = \frac{Rg}{v^2}$$

Velocity $v = (0.6 \text{ rev/s}) \left(\frac{2\pi R}{1 \text{ rev}} \right) = 0.6 \times \frac{2\pi \times 2.5}{1} = 9.43 \text{ m/s}$ and substituting it for v

the minimum coefficient of friction is $\mu_s = 0.28$.

P. 5.114: A 70 kg person rides in a 30 kg cart moving at 12 m/s at the top of the hill that is the shape of an arc of a circle with a radius of 40 m. (a) What is the apparent weight of the person as the cart passes over the top of the hill? (b) Determine the maximum speed that the cart may travel at the top of the hill without losing contact with the surface.



Force acting on the person at the top of the hill are his/her weight mg and the normal force N . The car is on a circular path and so is the person and the above two forces are centripetal forces, hence, by the second Newton law on a circular path we have

$$mg - N = \frac{mv^2}{R}$$

The apparent weight of the person is the normal force N acting on him/her. Thus

$$N = mg - \frac{mv^2}{R} = m \left(g - \frac{v^2}{R} \right) = 70 \times \left(9.8 - \frac{12^2}{40} \right) = 434 N$$

This can be compared to his/her natural weight $w = mg = 70 \times 9.8 = 686 N$

(b) The cart with the person on it at the top of the hill are subjected to forces of weight and the normal force. Thus, again, $Mg - N^* = \frac{Mv^2}{R}$ with M being the mass of the cart + person

For the cart to lose contact with the surface, this means that $N^* = 0$. The above equation then becomes

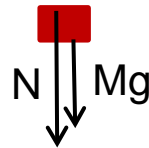
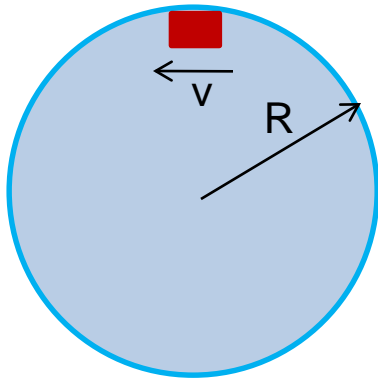
$$Mg = \frac{Mv^2}{R}$$

From which,

$$v_{\max} = \sqrt{Rg} = \sqrt{40 \times 9.8} = 19.8 m/s$$

Note, the maximum velocity is independent of the mass of the cart and person!

P. 5.118: A physics major is paying his tuition by performing in a traveling carnival. He rides a motorcycle inside a hollow transparent plastic sphere. After gaining sufficient speed, he travels in a vertical circle with a radius of 13 m. The physics major has mass 70 kg and his motorcycle has mass 40 kg. (a) What minimum speed must he have at the top of the circle if the tires of the motorcycle are not to lose contact with the sphere? (b) At the bottom of the circle his speed is twice the value calculated in part (a). What is the magnitude of the normal force exerted on the motorcycle by the sphere at this point?



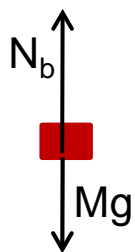
Forces N and mg are centripetal forces responsible for the circular motion, hence,

$$N + Mg = \frac{Mv^2}{R}$$

When the motorcycle is on the verge of losing contact with the sphere, the normal force N becomes zero, i.e.,

$$v = \sqrt{Rg} = \sqrt{13 \times 9.8} = 11.3 \text{ m/s}$$

(b) At the bottom of the circle, his speed is $2v = 22.6 \text{ m/s}$ and the forces acting are as shown:



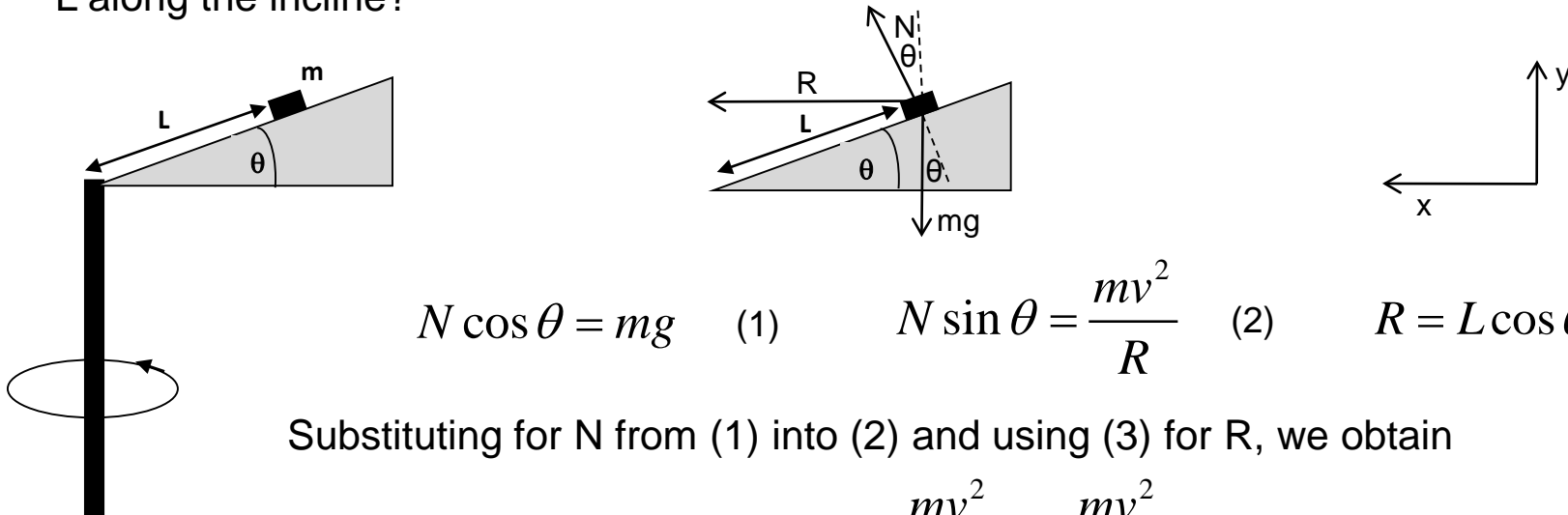
Again, these are centripetal forces, hence,

$$N_b - Mg = \frac{M(2v)^2}{R}$$

The normal force at the bottom of the circle is thus

$$N_b = Mg + \frac{M(2v)^2}{R} = 110 \times 9.8 + \frac{110 \times (22.6)^2}{13} = 5400 \text{ N}$$

A small block of mass m is placed on a very smooth wedge that has an acute angle θ with the horizontal. The wedge is firmly attached to a rod that is spun at a constant rate of rotation. How many revolutions per second must the block make so as it “stays” a distance L along the incline?



$$N \cos \theta = mg \quad (1) \quad N \sin \theta = \frac{mv^2}{R} \quad (2) \quad R = L \cos \theta \quad (3)$$

Substituting for N from (1) into (2) and using (3) for R , we obtain

$$mg \tan \theta = \frac{mv^2}{R} = \frac{mv^2}{L \cos \theta}$$

Solving for v we get

$$v = \sqrt{gL \tan \theta \cos \theta} = \sqrt{Lg \sin \theta}$$

Number of revolutions per second (frequency f) is inverse of the period of revolution T , hence

$$f = \frac{1}{T} = \frac{1}{\frac{2\pi R}{v}} = \frac{v}{2\pi R} = \frac{\sqrt{Lg \sin \theta}}{2\pi L \cos \theta} = \frac{1}{2\pi} \sqrt{\frac{g \sin \theta}{L \cos^2 \theta}}$$