This print-out should have 6 questions. Multiple-choice questions may continue on the next column or page – find all choices before making your selection. The due time is Central time.

Chapter 5 problems.

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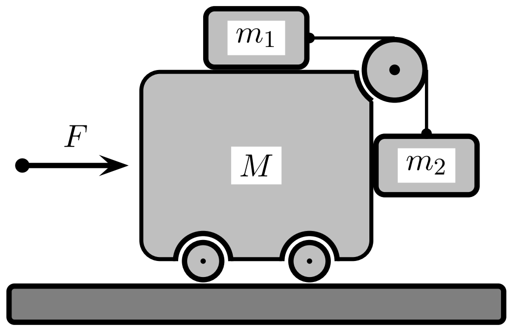
001 (part 1 of 1) 10 points

Assume: All surfaces, wheels, and pulley are frictionless. The inextensible cord and pulley are massless.



Given $M = 29.8$ kg, $m_1 = 4.29$ kg, $m_2 = 3.38$ kg, and $g = 9.8\ \text{m/s}^2$.

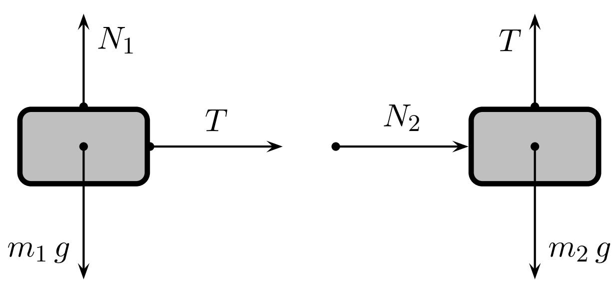
What horizontal force must be applied to the cart shown in the figure in order for the blocks to remain stationary relative to the cart?

Correct answer: 289.314 N.

Explanation:

Note: The blocks m_1 and m_2 being stationary relative to the cart M means that they have the same *non-zero* horizontal acceleration a relative to the ground.

Consider the free-body diagrams.



Applying Newton's second law to the horizontal motion of the m_1 block yields

$$m_1 : \quad \sum F_x = T = m_1\,a\,, \qquad (1)$$

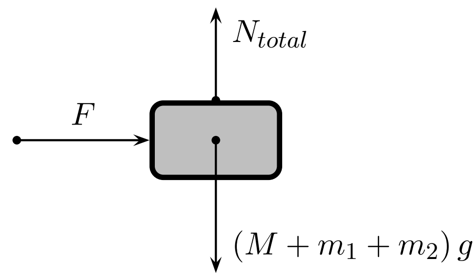
while for the vertical motion of the m_2 block we have

$$m_2 : \quad \sum F_y = T - m_2\,g = 0 \qquad (2)$$

because it accelerates horizontally but not vertically. Combining the above two Eqs. (1) & (2), and solving for the a yields

$$a = \frac{m_2}{m_1}\,g\,. \qquad (3)$$

On the other hand, a is related to the external force F via the equation of horizontal motion for the whole cart-plus-two-blocks system:



In light of the above whole-system diagram, the horizontal equation is simply

$$\sum F_x = F = (M + m_1 + m_2)a \qquad (4)$$

and hence

$$F = (M + m_1 + m_2) \times \frac{m_2}{m_1}\,a = 289.314\ \text{N}.$$

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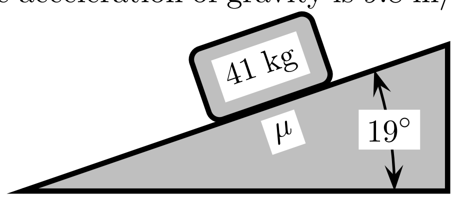
002 (part 1 of 3) 4 points

A block is at rest on the incline shown in the figure. The coefficients of static and kinetic friction are $\mu_s = 0.42$ and $\mu_k = 0.36$, respectively.

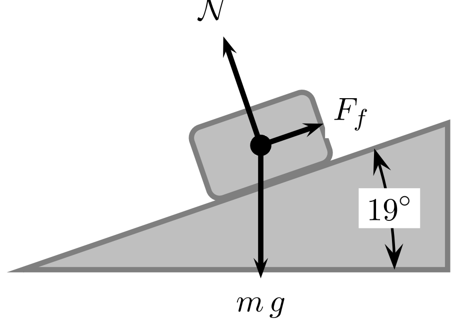
The acceleration of gravity is $9.8\ \text{m/s}^2$.



What is the frictional force acting on the 41 kg mass?

Correct answer: 130.813 N.

Explanation:



The forces acting on the block are shown in the figure. Since the block is at rest, the magnitude of the friction force should be equal to the component of the weight on the plane of the incline

$$\begin{aligned} F_f &= M\,g\,\sin\theta \\ &= (41\text{ kg})\,(9.8\text{ m/s}^2)\,\sin 19^\circ \\ &= 130.813\text{ N}\,. \end{aligned}$$

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003 (part 2 of 3) 3 points

What is the largest angle which the incline can have so that the mass does not slide down the incline?

Correct answer: 22.7824 °.

Explanation:

The largest possible value the static friction force can have is $F_{f,max} = \mu_s \mathcal{N}$, where the normal force is $\mathcal{N} = M g \cos\theta$. Thus, since $F_f = M\,g\,\sin\theta$,

$$\begin{aligned} M\,g\,\sin\theta_m &= \mu_s M\,g\,\cos\theta_m \\ \tan\theta_m &= \mu_s \\ \theta_m &= \tan^{-1}(\mu_s) \\ &= \tan^{-1}(0.42) \\ &= 22.7824^\circ\,. \end{aligned}$$

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004 (part 3 of 3) 3 points

What is the acceleration of the block down the incline if the angle of the incline is 30° ?

Correct answer: 1.84466 m/s^2.

Explanation:

When θ exceeds the value found in part 2, the block starts moving and the friction force is the kinetic friction

$$F_k = \mu_k \mathcal{N} = \mu_k\,M\,g\cos\theta.$$

Newton's equation for the block then becomes

$$\begin{aligned} M\,a &= M\,g\,\sin\theta - F_f \\ &= M\,g\,\sin\theta - \mu_k\,M\,g\,\cos\theta \end{aligned}$$

and

$$\begin{aligned} a &= g\,[\sin\theta - \mu_k\,\cos\theta] \\ &= (9.8\text{ m/s}^2)\,[\sin 30^\circ - (0.36)\,\cos 30^\circ] \\ &= 1.84466\text{ m/s}^2\,. \end{aligned}$$

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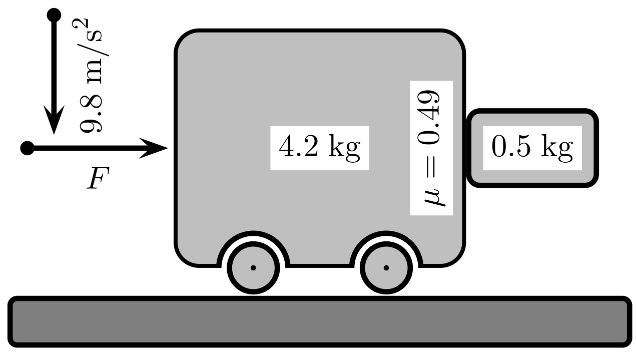
005 (part 1 of 1) 10 points

Static friction 0.49 between a 0.5 kg block and a 4.2 kg cart. There is no kinetic friction between the cart and the horizontal surface.

The acceleration of gravity is 9.8 m/s^2 .

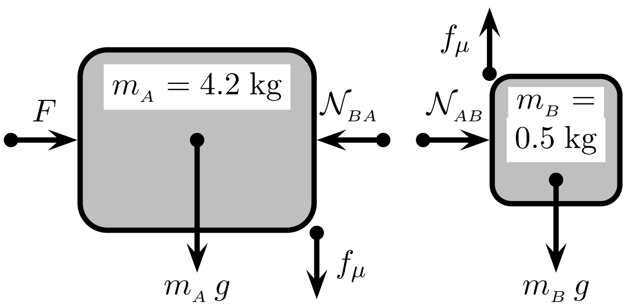


What minimum force F must be exerted on the 4.2 kg cart in order for the 0.5 kg block not to fall?

1. $F = 94$ N **correct**
2. $F = 84$ N
3. $F = 64$ N
4. $F = 49$ N
5. $F = 80$ N
6. $F = 92$ N
7. $F = 66$ N
8. $F = 86$ N
9. $F = 91$ N
10. $F = 68$ N

Explanation:

$$\begin{aligned} \text{Let}: \quad m_A &= 4.2\text{ kg}\,, \quad \text{Cart } \mathbf{A} \\ m_B &= 0.5\text{ kg}\,, \quad \text{Block } \mathbf{B} \\ \mu_{AB} &= 0.49\,, \quad \text{between } \mathbf{A} \text{ and } \mathbf{B} \\ \mu_k &= 0\,, \quad \text{horizontal surface, and} \\ g &= 9.8\text{ m/s}^2\,. \end{aligned}$$



The condition that block B not fall implies that its vertical acceleration is zero. Applying Newton's second law for B in the horizontal and vertical directions yields

$$\sum F_x = \mathcal{N} = m_B\, a_x \qquad (1)$$

$$\sum F_y = \mu_{AB}\,\mathcal{N} - m_B\, g = 0\,, \qquad (2)$$

where $\mathcal{N}$ is the normal force exerted on block B by cart A. Applying Newton's law on cart A in the horizontal direction we have

$$F - \mathcal{N} = m_A\, a_x\,.$$

Solving for F,

$$F = \mathcal{N} + m_A\, a_x\,. \qquad (3)$$

The normal force $\mathcal{N}$ and the acceleration a_x may be determined from the first two equations. From Eq. 2 we find

$$\mathcal{N} = \frac{m_B\, g}{\mu_{AB}}\,.$$

From Eq. 1 and 2, and solving for a_x yields

$$a_x = \frac{\mathcal{N}}{m_B} = \frac{g}{\mu_{AB}}$$

Since we now have expressions for $\mathcal{N}$ and a_x, we can find the force F required so that block B does not fall. From Eq. 3 again

$$\begin{aligned} F &= \mathcal{N} + m_A\, a_x \\ &= m_B\,\frac{g}{\mu_{AB}} + m_A\,\frac{g}{\mu_{AB}} \\ &= (4.2\text{ kg} + 0.5\text{ kg})\,\frac{9.8\text{ m/s}^2}{0.49} \\ &= 94\text{ N}\,. \end{aligned}$$

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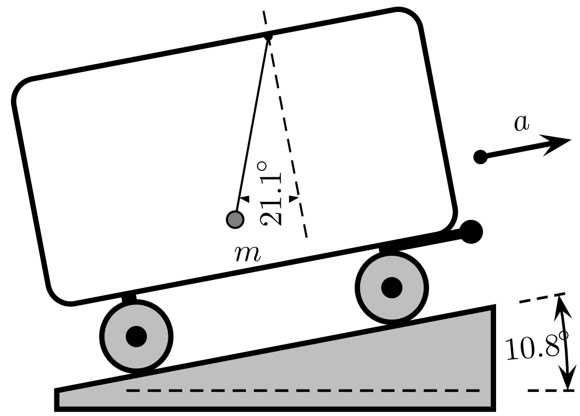
006 (part 1 of 1) 10 points

Given: $g = 9.8\text{ m/s}^2$.

Consider a boxcar accelerating *up* a 10.8° slope. Inside the boxcar, an object of unknown mass m hangs on a string attached to the boxcar's ceiling.

When the car accelerates uphill at a *steady* rate a, the string hangs at a constant angle $\theta = 21.1°$ from the perpentidular to the boxcar's ceiling and floor.



Given the angles $\theta = 21.1° > \phi = 10.8°$, calculate the boxcar's acceleration a.

Correct answer: $1.87819\ \text{m/s}^2$.

Explanation:

Basic Concept: In a non-inertial frame such as an accelerating boxcar, the inertial force $-m\vec{a}$ combines with the true gravitational force $m\vec{g}$ into a single *apparent weight* force

$$\vec{\mathcal{W}}_{\text{app}} \;=\; m(\vec{g} - \vec{a}_{\text{frame}}). \qquad (1)$$

In fact, the Equivalence Principle says that there is no observable difference between the true gravity and the inertial forces, so in a non-inertial frame there is a *net effective gravity*

$$\vec{g}_{\text{eff}} \;=\; \vec{g} \;-\; \vec{a}_{\text{frame}}. \qquad (2)$$

Solution: Consider the hanging object in the non-inertial frame of the accelerating boxcar. In the car's frame, the objects hangs without motion so its *apparent weight* (1) must be balanced by the string's tension. Hence, the direction of the *effective gravity* (2) must be opposite to the string's pull on the object, which is 10.8° from the perpendicular to the boxcar's floor and ceiling and $21.1° - 10.8° = 10.3°$ from the true vertical.

At this point, the problem reduces to geometry: Given the directions of vectors $\vec{g}$, $\vec{a}$ and $\vec{g} - \vec{a}$ and the magnitude $g = 9.8$ m/s^2, find the magnitude a. We can solve this question using the sine theorem, but it is just as easy to solve in Carthesian coordinates. Let the x axis run uphull along the boxcars's floor while the y axis is perpendicular the the floor: In these coordinates,

$$a_x = a, \quad a_y = 0, \tag{3}$$

$$g_x = -g \sin\phi, \quad g_y = -g \cos\phi \tag{4}$$

where $\phi = 10.8^\circ$ is the hill's slope. At the same time, the string's direction indicates

$$g_x^{\text{eff}} = -g^{\text{eff}} \sin\theta, \quad g_y^{\text{eff}} = -g^{\text{eff}} \cos\theta \tag{5}$$

where $\theta = 21.1^\circ$ is the angle between the string and the y axis. Consequently,

$$\begin{aligned} \tan\theta &= \frac{g_x^{\text{eff}}}{g_y^{\text{eff}}} = \frac{g_x - a_x}{g_y - a_y} \\ &= \frac{-g\sin\phi - a}{-g\cos\phi} \\ &= \tan\phi + \frac{a}{g\cos\phi} \end{aligned} \tag{6}$$

and therefore

$$\begin{aligned} a &= (\tan\theta - \tan\phi) \times g\cos\phi \\ &= 1.87819 \text{ m/s}^2. \end{aligned} \tag{7}$$