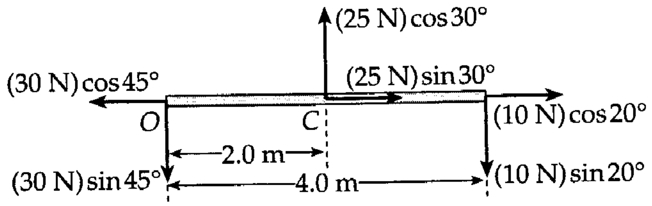
SOLUTIONS TO SELECTED END-OF-CHAPTER PROBLEMS

3. Calculate the net torque (magnitude and direction) on the beam in Figure P8.3 about (a) an axis through O perpendicular to the page and (b) an axis through C perpendicular to the page.

Solution

To solve this problem in the easiest manner, first resolve all of the forces shown in Figure P8.3 into components parallel to and perpendicular to the beam as shown below. Then, it is observed that most of these components have zero lever arms about an axis perpendicular to the page through point O; and the lever arms that are not zero are easily determined. This is also true for a similar axis through point C.



(a) $\tau_O = +\left[(25\text{ N})\cos 30°\right](2.0\text{ m}) - \left[(10\text{ N})\sin 20°\right](4.0\text{ m}) = +30\text{ N}\cdot\text{m}$

or $\tau_O = 30\text{ N}\cdot\text{m}$ counterclockwise ◊

(b) $\tau_C = +\left[(30\text{ N})\sin 45°\right](2.0\text{ m}) - \left[(10\text{ N})\sin 20°\right](2.0\text{ m}) = +36\text{ N}\cdot\text{m}$

or $\tau_C = 36\text{ N}\cdot\text{m}$ counterclockwise ◊

9. A cook holds a 2.00-kg carton of milk at arm's length (Fig. P8.9). What force $\vec{\mathbf{F}}_B$ must be exerted by the biceps muscle? (Ignore the weight of the forearm.)

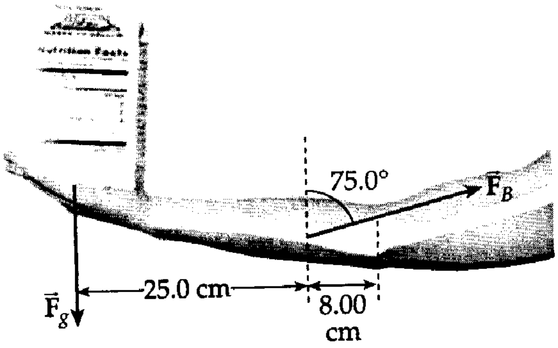


Figure P8.9

Solution

For the system consisting of the forearm and the milk carton to be in equilibrium, it is necessary that the sum of the torques be zero about any axis we choose.

The upper arm exerts some unknown force on the forearm at the elbow. However, if we select a rotation axis that is perpendicular to the page and passes through the elbow, this unknown force will have zero lever arm and exert zero torque about our axis.

Resolve the force exerted by the biceps muscle into horizontal and vertical components:

$$(F_B)_x = F_B \sin 75.0^\circ \qquad \text{and} \qquad (F_B)_y = F_B \cos 75.0^\circ$$

The line along which the horizontal component acts passes through the chosen rotation axis, so that component exerts zero torque. Applying the second condition for equilibrium to the system (forearm and milk carton) yields

$$\sum \tau = +F_g\,(25.0\text{ cm} + 8.00\text{ cm}) - (F_B)_y\,(8.00\text{ cm}) = 0$$

or $$mg\,(25.0\text{ cm} + 8.00\text{ cm}) - (F_B \cos 75.0^\circ)(8.00\text{ cm}) = 0$$

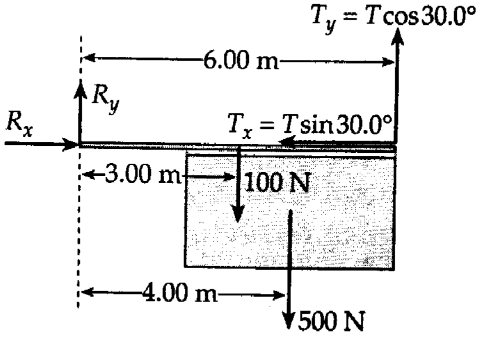
The force exerted by the biceps muscle is then given by

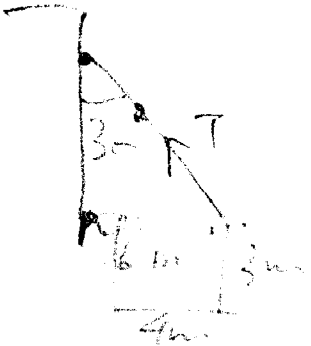
$$F_B = \frac{\left[(2.00\text{ kg})(9.80\text{ m/s}^2)\right](33.0\text{ cm})}{(8.00\text{ cm})\cos 75.0^\circ} = 312\text{ N} \qquad \Diamond$$

17. A 500-N uniform rectangular sign 4.00 m wide and 3.00 m high is suspended from a horizontal, 6.00-m-long, uniform, 100 N rod as indicated in Figure P8.17. The left end the rod is supported by a hinge, and the right end is supported by a thin cable making a 30.0° angle with the vertical. (a) Find the tension T in the cable. (b) Find the horizontal and vertical components of force exerted on the left end of the rod by the hinge.

Solution

The free-body diagram of the sign-rod combination is given below. Note that the tension in the cable and the reaction force exerted on the left end of the rod by the hinge have been resolved into horizontal and vertical components.





Consider a rotation axis perpendicular to the page and passing through the left end of the rod and apply the second condition of equilibrium, $\Sigma\tau = 0$, to this system. This gives

$$+(T\cos 30.0°)(6.00\text{ m}) - (100\text{ N})(3.00\text{ m}) - (500\text{ N})(4.00\text{ m}) = 0$$

or $$T = \frac{2.30\times10^3\text{ N}\cdot\text{m}}{(6.00\text{ m})\cos 30.0°} = 443\text{ N} \qquad \Diamond$$

Now, apply the first condition of equilibrium to this system:

$$\Sigma F_x = 0 \Rightarrow R_x - T\sin 30.0° = 0$$

or $$R_x = +(443\text{ N})\sin 30.0° = +222\text{ N} = 222\text{ N toward the right} \qquad \Diamond$$

and $$\Sigma F_y = 0 \Rightarrow R_y + T\cos 30.0° - 100\text{ N} - 500\text{ N} = 0$$

or $$R_y = 600\text{ N} - (443\text{ N})\cos 30.0° = +216\text{ N} = 216\text{ N upward} \qquad \Diamond$$

27. The large quadriceps muscle in the upper leg terminates at its lower end in a tendon attached to the upper end of the tibia (Fig. P8.27a). The forces on the lower leg when the leg is extended are modeled as in Figure P8.27b, where $\vec{\mathbf{T}}$ is the force of tension in the tendon, $\vec{\mathbf{w}}$ is the force of gravity acting on the lower leg, and $\vec{\mathbf{F}}$ is the force of gravity acting on the foot. Find $\vec{\mathbf{T}}$ when the tendon is at an angle of 25.0° with the tibia, assuming that w = 30.0 N, F = 12.5 N, and the leg is extended at an angle θ of 40.0° with the vertical. Assume that the center of gravity of the lower leg is at its center and that the tendon attaches to the lower leg at a point one-fifth of the way down the leg.

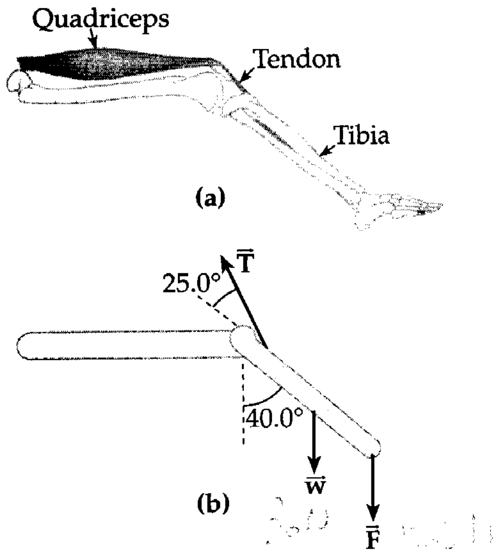
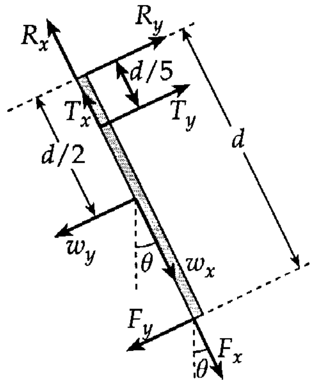


Figure P8.27

Solution

The free-body diagram of the tibia is given at the right. Note that $\theta = 40.0°$ and that all forces have been resolved into components parallel to and perpendicular to the tibia. We shall choose an axis that is perpendicular to the page and passing through the upper end of the tibia. Only T_y, w_y, and F_y have non-zero torques about this axis. The magnitudes of these components are:



$$T_y = T\sin 25.0°$$

$$w_y = w\sin 40.0° = (30.0\text{ N})\sin 40.0° = 19.3\text{ N}$$

and $F_y = F\sin 40.0° = (12.5\text{ N})\sin 40.0° = 8.03\text{ N}$

When the tibia is held in rotational equilibrium in the position shown, the second condition of equilibrium ($\Sigma\tau = 0$) gives the tension in the tendon as:

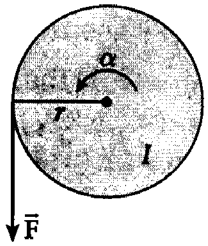
$$+(T\sin 25.0°)\frac{d}{5} - (19.3\text{ N})\frac{d}{2} - (8.03\text{ N})d = 0$$

or $T = 5\left(\dfrac{17.7\text{ N}}{\sin 25.0°}\right) = 209\text{ N}$ ◊

35. A 150-kg merry-go-round in the shape of a uniform, solid, horizontal disk of radius 1.50 m is set in motion by wrapping a rope about the rim of the disk and pulling on the rope. What constant force must be exerted on the rope to bring the merry-go-round from rest to an angular speed of 0.500 rev/s in 2.00 s?

Solution

The sketch at the right gives a view of the merry-go-round from above. The moment of inertia of a uniform, solid disk of mass $M = 150$ kg an radius $r = 1.50$ m is:



$$I = \frac{1}{2}Mr^2 = \frac{1}{2}(150 \text{ kg})(1.5 \text{ m})^2 = 169 \text{ kg}\cdot\text{m}^2$$

The desired angular acceleration for the merry-go-round is:

$$\alpha = \frac{\omega - \omega_0}{\Delta t} = \frac{0.500 \text{ rev/s} - 0}{2.00 \text{ s}} = 0.250 \ \frac{\text{rev}}{\text{s}^2}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 1.57 \text{ rad/s}^2$$

We take counterclockwise torques (and hence, angular accelerations) as positive, and use the rotational form of Newton's second law to find the tension in the rope required to produce this angular acceleration:

$$\Sigma\tau = Fr = I\alpha \quad \text{or} \quad F = \frac{I\alpha}{r} = \frac{(169 \text{ kg}\cdot\text{m})^2(1.57 \text{ rad/s}^2)}{1.50 \text{ m}} = 177 \text{ N} \qquad \Diamond$$

39. A 10.0-kg cylinder rolls without slipping on a rough surface. At the instant when its center of gravity has a speed of 10.0 m/s, determine (a) the translational kinetic energy of its center of gravity, (b) the rotational kinetic energy about its center of gravity, and (c) its total kinetic energy.

Solution

(a) The translational kinetic energy of the cylinder is given by $KE_t = \frac{1}{2}mv^2$, where v is the translational speed of the center of gravity. Thus,

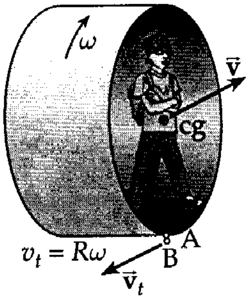
$KE_t = \frac{1}{2}(10.0 \text{ kg})(10.0 \text{ m/s})^2$ or $KE_t = 500 \text{ J}$ ◊

(b) The rotational kinetic energy of the cylinder is $KE_r = \frac{1}{2}I\omega^2$ where I is the moment of inertia about the axis through its center of gravity, and ω is the angular speed about this axis. Assuming a uniform, solid cylinder of radius R,

$$I = \frac{1}{2}mR^2$$

and the rotational kinetic energy is $KE_r = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega^2 = \frac{1}{4}m(R\omega)^2$

The product $R\omega$ is the same as the tangential speed of a point on the rim of the cylinder, $v_t = R\omega$, so $KE_r = \frac{1}{4}mv_t^2$



If the wheel rolls without slipping, the tangential speed of a point on the rim is the same as the translational speed of the center of gravity. To understand why this is true, imagine yourself to be at the center of the wheel and moving to your left at speed v. Looking down, you would see point A at the wheel's rim moving to your right with the tangential speed $v_t = R\omega$. You would also see point B on the ground moving to your right at the speed v, the speed of the center of gravity (and you) relative to the ground.

Now, if the rim of the wheel is not slipping against the ground, the points A and B (in contact with each other) must move at the same speed. Thus, it is necessary that $v = v_t = R\omega$ if slipping does not occur. The rotational kinetic energy of the wheel is therefore,

$KE_r = \frac{1}{4}mv_t^2 = \frac{1}{4}mv^2 = \frac{1}{4}(10.0 \text{ kg})(10.0 \text{ m/s})^2 = 250 \text{ J}$ ◊

(c) The total kinetic energy of the rolling wheel is then:

$KE_{\text{total}} = KE_t + KE_r = 500 \text{ J} + 250 \text{ J} = 750 \text{ J}$ ◊

43. The top in Figure P8.43 has a moment of inertia of $4.00 \times 10^{-4}\ \text{kg}\cdot\text{m}^2$ and is initially at rest. It is free to rotate about a stationary axis, *AA′*. A string wrapped around a peg along the axis of the top is pulled in such a manner as to maintain a constant tension of 5.57 N in the string. If the string does not slip while wound around the peg, what is the angular speed of the top after 80.0 cm of string has been pulled off the peg? [**Hint:** Consider the work done.]

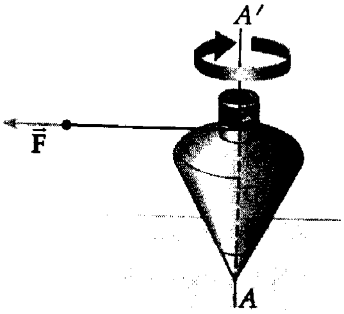


Figure P8.43

Solution

The agent pulling on the end of the string does work on the system consisting of the string and top. When the end of the string has been moved distance *s*, the work done on this system is $W_{net} = Fs\cos 0°$, where *F* is the constant tension maintained in the string.

Neglecting the mass of the string, the only kinetic energy of the system is the rotational kinetic energy of the top. The work-energy theorem, $W_{net} = KE_f - KE_i$, then becomes

$$Fs\cos 0° = \tfrac{1}{2} I_{\text{top}} \omega_f^2 - \tfrac{1}{2} I_{\text{top}} \omega_i^2$$

If the top starts from rest and the tension maintained in the string is $F = 5.57\ \text{N}$, the angular speed of the top after the end of the string has been moved 80.0 cm is

$$\omega_f = \sqrt{\frac{2Fs\cos 0°}{I_{\text{top}}}} = \sqrt{\frac{2(5.57\ \text{N})(0.800\ \text{m})(1)}{4.00\times10^{-4}\ \text{kg}\cdot\text{m}^2}} = 149\ \text{rad/s} \qquad \Diamond$$

53. A 60.0-kg woman stands at the rim of a horizontal turntable having a moment of inertia of 500 kg·m^2 and a radius of 2.00 m. The turntable is initially at rest and is free to rotate about a frictionless, vertical axle through its center. The woman then starts walking around the rim clockwise (as viewed from above the system) at a constant speed of 1.50 m/s relative to the Earth. (a) In what direction and with what angular speed does the turntable rotate? (b) How much work does the woman do to set herself and the turntable into motion?

Solution

(a) If no external agent exerts a torque about the vertical axis through the center of the turntable, the total angular momentum of the system (woman plus turntable) about this axis remains constant. When the woman is at rest on the stationary turntable, the total angular momentum of the system is zero $(L_i = 0)$. As the woman starts walking around the axis, the turntable must develop a counterclockwise angular momentum whose magnitude equals the magnitude of the woman's clockwise angular momentum. These two contributions to the total angular momentum will then cancel each other.

Treating the woman as a point object on the rim of the turntable (at distance $r = 2.00\text{ m}$ from the axis), her moment of inertia about the central axis is $I_w = m_w r^2$. Taking counterclockwise angular momentum as positive, the woman's clockwise angular momentum as she walks at constant speed v relative to Earth is

$$L_w = -I_w \omega_w = -\left(m_w r^2\right)\left(\frac{v}{r}\right) = -m_w r v$$

The angular momentum of the turntable is $L_t = I_t \omega_t$ and conservation of angular momentum requires that

$$L_{final} = \left(L_t + L_w\right) = L_i = 0 \qquad \text{or} \qquad I_t \omega_t - m_w r v = 0$$

Thus $\omega_t = \dfrac{m_w r v}{I_t} = \dfrac{(60.0\text{ kg})(2.00\text{ m})(1.50\text{ m/s})}{500\text{ kg}\cdot\text{m}^2} = +0.36\text{ rad/s}$

or $\omega_t = 0.36\text{ rad/s}$ counterclockwise ◊

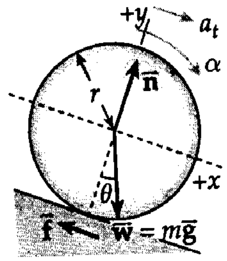
(b) The work-energy theorem gives the work done by the woman to set this system in motion as

$$W_{net} = KE_f - KE_i = \left(\tfrac{1}{2} I_t \omega_t^2 + \tfrac{1}{2} m_w v^2\right) - 0$$

$$W_{net} = \frac{1}{2}\left(500\text{ kg}\cdot\text{m}^2\right)(0.36\text{ rad/s})^2 + \frac{1}{2}(60.0\text{ kg})(1.50\text{ m/s})^2 = 99.9\text{ J}$$ ◊

13. A solid 2.0-kg ball of radius 0.50 m starts at a height of 3.0 m above the surface of the Earth and rolls down a 20° slope. A solid disk and a ring start at the same time and the same height. The ring and disk each have the same mass and radius as the ball. Which of the three wins the race to the bottom if all roll without slipping?

Solution



Consider the free-body diagram of an object rolling down the incline. The object could be either a solid sphere (ball), a solid cylinder (disk), or a hoop (ring). If the object rolls without slipping, then $a_t = r\alpha$ where a_t is the linear acceleration of the center of gravity and α is the angular acceleration about the rotation axis.

From $\sum F_x = ma_x$ we obtain $\quad mg\sin\theta - f = ma_t \quad$ [1]

Now, consider an axis perpendicular to the page and passing through the center of the object.

$\sum\tau = I\alpha$ becomes $\quad f \cdot r = I\alpha = I\left(\frac{a_t}{r}\right) \quad$ or $\quad f = \left(\frac{I}{r^2}\right)a_t$

Substitute this result into Equation [1] and simplify to obtain $\quad a_t = \dfrac{g\sin\theta}{\left(1 + I/mr^2\right)}$

as the linear acceleration of the center of gravity of the object.

For a solid sphere, $\quad I = \frac{2}{5}mr^2 \quad$ so $\quad a_{\text{sphere}} = \dfrac{g\sin\theta}{1.4}$

For a solid cylinder, $\quad I = \frac{1}{2}mr^2 \quad$ so $\quad a_{\text{cylinder}} = \dfrac{g\sin\theta}{1.5}$

Finally, for a hoop, $\quad I = mr^2 \quad$ so $\quad a_{\text{ring}} = \dfrac{g\sin\theta}{2.0}$

Thus, we find $a_{\text{sphere}} > a_{\text{cylinder}} > a_{\text{ring}}$, so the sphere wins the race, the disk comes in second, and the ring is last. ◇

Note that each of the calculated accelerations is independent of the both the mass and the radius of the object. Thus, the three objects need not be of the same mass or size. Only the shapes and the distribution of the mass within the objects affect the outcome of the race.

66. (a) Without the wheels, a bicycle frame has a mass of 8.44 kg. Each of the wheels can be roughly modeled as a uniform solid disk with a mass of 0.820 kg and a radius of 0.343 m. Find the kinetic energy of the whole bicycle when it is moving forward at 3.35 m/s. (b) Before the invention of a wheel turning on an axle, ancient people moved heavy loads by placing rollers under them. (Modern people use rollers, too: Any hardware store will sell you a roller bearing for a lazy Susan.) A stone block of mass 844 kg moves forward at 0.335 m/s, supported by two uniform cylindrical tree trunks, each of mass 82.0 kg and radius 0.343 m. There is no slipping between the block and the rollers or between the rollers and the ground. Find the total kinetic energy of the moving objects.

Solution

(a) The frame and the center of each wheel move forward at $v = 3.35$ m/s and each wheel also turns at angular speed $\omega = v/R$. The total kinetic energy of the bicycle is $KE = KE_t + KE_r$, or

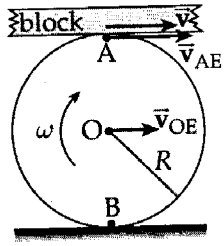
$$KE = \frac{1}{2}\left(m_{frame} + 2m_{wheel}\right)v^2 + 2\left(\frac{1}{2}I_{wheel}\,\omega^2\right)$$

$$= \frac{1}{2}\left(m_{frame} + 2m_{wheel}\right)v^2 + \frac{1}{2}\left(m_{wheel}R^2\right)\left(\frac{v^2}{R^2}\right)$$

This yields

$$KE = \frac{1}{2}\left(m_{frame} + 3m_{wheel}\right)v^2 = \frac{1}{2}\left[8.44\text{ kg} + 3(0.820\text{ kg})\right](3.35\text{m/s})^2 = 61.2\text{ J}$$ ◊

(b) Since the block does not slip on the roller, its forward speed must equal that of point A, the uppermost point on the rim of the roller. That is, $v = \left|\vec{\mathbf{v}}_{AE}\right|$ where $\vec{\mathbf{v}}_{AE}$ is the velocity of A relative to Earth.



$\left|\vec{\mathbf{v}}_{OE}\right| = v_O = R\omega$

Please refer to the discussion in Part (b) of Problem 39 earlier in this chapter. Since the roller does not slip on the ground, the velocity of point O (the roller center) must have the same magnitude as the tangential speed of point B (the point on the roller rim in contact with the ground). That is, $\left|\vec{\mathbf{v}}_{OE}\right| = R\omega = v_O$. Also, note that the velocity of point A relative to the roller center has a magnitude equal to the tangential speed $R\omega$, or $\left|\vec{\mathbf{v}}_{AO}\right| = R\omega = v_O$.

From the discussion of relative velocities in Chapter 3, we know that $\vec{\mathbf{v}}_{AE} = \vec{\mathbf{v}}_{AO} + \vec{\mathbf{v}}_{OE}$. Since all of these velocities are in the same direction, we may add their magnitudes getting $\left|\vec{\mathbf{v}}_{AE}\right| = \left|\vec{\mathbf{v}}_{AO}\right| + \left|\vec{\mathbf{v}}_{OE}\right|$, or $v = v_O + v_O = 2v_O = 2R\omega$. Thus, we have determined that the translational speed of the center of the rollers (trees) is one half the speed of the stone $(v_O = v/2)$. Then, the angular speed of the roller is $\omega = v_O/R = v/2R$.

The total kinetic energy is $KE = KE_{translation} + KE_{rotation}$, or

$$KE = \frac{1}{2}m_{stone}v^2 + 2\left[\frac{1}{2}m_{tree}\left(\frac{v}{2}\right)^2\right] + 2\left(\frac{1}{2}I_{tree}\omega^2\right)$$

$$= \left(\frac{1}{2}m_{stone} + \frac{1}{4}m_{tree}\right)v^2 + \frac{1}{2}m_{tree}R^2\left(\frac{v^2}{4R^2}\right) = \frac{1}{2}\left(m_{stone} + \frac{3}{4}m_{tree}\right)v^2$$

This gives $KE = \frac{1}{2}\left[844\text{ kg} + \frac{3}{4}(82.0\text{ kg})\right](0.335\text{ m/s})^2 = 50.8\text{ J}$ ◊

73. A uniform solid cylinder of mass M and radius R rotates on a frictionless horizontal axle (Fig. P8.73). Two objects with equal masses hang from light cords wrapped around the cylinder. If the system is released from rest, find (a) the tension in each cord and (b) the acceleration of each object after the objects have descended a distance of h.

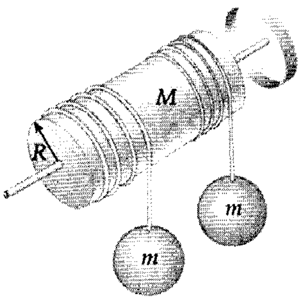
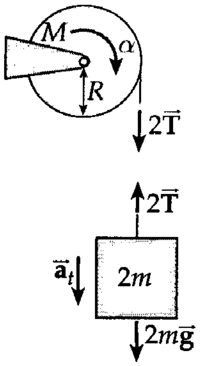


Figure P8.73

Solution

In the free-body diagrams at the right, the two falling masses have been combined into one object of mass $2m$. The total downward force these masses exert on the cylinder is $2T$ where T is the tension in either cord. If the cords do not slip on the cylinder, the linear acceleration of each falling object is related to the angular acceleration of the cylinder by $a_t = R\alpha$.



Choose an axis perpendicular to the page and passing through the center of the cylinder. Then, applying $\Sigma\tau = I\alpha$ to the cylinder gives

$$(2T)R = \left(\tfrac{1}{2}MR^2\right)\alpha = \left(\tfrac{1}{2}MR^2\right)\left(\frac{a_t}{R}\right) \qquad \text{or} \qquad T = \tfrac{1}{4}Ma_t \qquad [1]$$

Now apply $\Sigma F_y = ma_y$ to the falling objects to obtain

$$(2m)g - 2T = (2m)a_t \qquad \text{or} \qquad a_t = g - \frac{T}{m} \qquad [2]$$

(a) Substituting Equation [2] into Equation [1] yields

$$T = \frac{Mg}{4} - \left(\frac{M}{4m}\right)T \qquad \text{which reduces to} \qquad T = \frac{Mmg}{M+4m} \qquad \Diamond$$

(b) From Equation [2], the acceleration of each falling object is found to be

$$a_t = g - \frac{1}{m}\left(\frac{Mmg}{M+4m}\right) = g - \frac{Mg}{M+4m} = \frac{4mg}{M+4m} \qquad \Diamond$$

Note that the results are independent of the distance fallen, h.

0. **A string is wrapped around a uniform cylinder of mass M and radius R. The cylinder is released from rest with the string vertical and its top end tied to a fixed bar (Fig. P8.80). Show that (a) the tension in the string is one-third the weight of the cylinder, (b)** the magnitude of the acceleration of the center of gravity is $2g/3$, and (c) the speed of the center of gravity is $(4gh/3)^{1/2}$ after the cylinder has descended through distance h. Verify your answer to (c) with the energy approach.

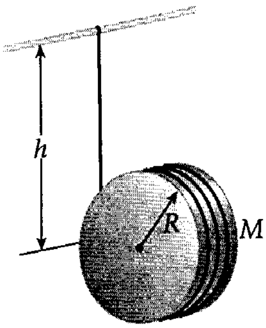
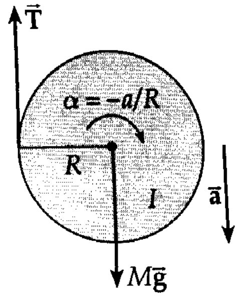


Figure P8.80

Solution



(a) The cylinder will rotate about an axis perpendicular to the page and through its center of gravity as the center of gravity accelerates downward. Applying Newton's second law to both the translational motion and the rotational motion gives:

From $\Sigma F_y = ma_y$, $\quad T - Mg = M(-a)$

or $\quad T = M(g-a)$ $\quad$ [1]

From $\Sigma\tau = I\alpha$, $\quad -TR = \left(\frac{1}{2}MR^2\right)\left(-\frac{a}{R}\right)$

or $\quad a = \frac{2T}{M}$ $\quad$ [2}

Substituting Equation [2] into Equation [1] and solving for the tension yields

$$T = Mg/3$$ ◊

(b) From Equation [2], $\quad a = \frac{2T}{M} = \frac{2(Mg/3)}{M} = 2g/3$ ◊

(c) From $v_y^2 = v_{0y}^2 + 2a_y\Delta y$, $\quad v = \sqrt{0 + 2\left(-\frac{2g}{3}\right)(-h)} = \sqrt{\frac{4gh}{3}}$ ◊

Using the work-energy theorem, $W_{net} = KE_f - KE_i$, we have

$$Mgh - Th = \frac{1}{2}Mv^2 - 0$$

or $\quad v = \sqrt{\frac{2Mgh - 2(Mg/3)h}{M}} = \sqrt{\frac{4gh}{3}}$ ◊