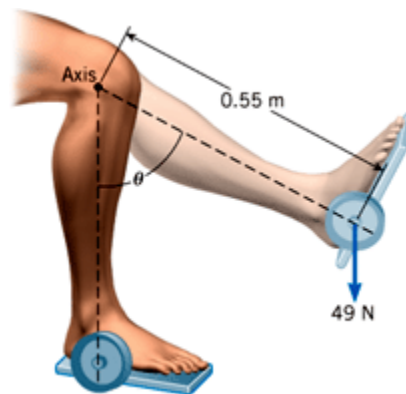


Week 8 homework

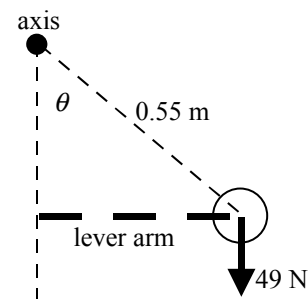
IMPORTANT NOTE ABOUT WEBASSIGN:

In the WebAssign versions of these problems, various details have been changed, so that the answers will come out differently. The method to find the solution is the same, but you will need to repeat part of the calculation to find out what your answer should have been.

WebAssign Problem 1: The drawing shows a lower leg being exercised. It has a 49-N weight attached to the foot and is extended at an angle θ with respect to the vertical. Consider a rotational axis at the knee. (a) When $\theta = 90.0^\circ$, find the magnitude of the torque that the weight creates. (b) At what angle θ does the magnitude of the torque equal $15 \text{ N} \cdot \text{m}$?



REASONING In both parts of the problem, the magnitude of the torque is given by Equation 9.1 as the magnitude F of the force times the lever arm ℓ . In part (a), the lever arm is just the distance of 0.55 m given in the drawing. However, in part (b), the lever arm is less than the given distance and must be expressed using trigonometry as $\ell = (0.55 \text{ m}) \sin \theta$. See the drawing at the right.



SOLUTION

a. Using Equation 9.1, we find that

$$\text{Magnitude of torque} = F\ell = (49 \text{ N})(0.55 \text{ m}) = \boxed{27 \text{ N} \cdot \text{m}}$$

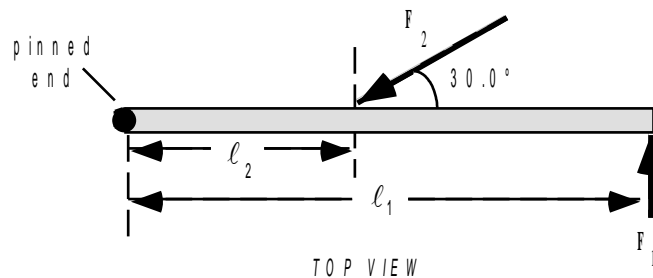
b. Again using Equation 9.1, this time with a lever arm of $\ell = (0.55 \text{ m}) \sin \theta$, we obtain

$$\text{Magnitude of torque} = 15 \text{ N} \cdot \text{m} = F\ell = (49 \text{ N})(0.55 \text{ m}) \sin \theta$$

$$\sin \theta = \frac{15 \text{ N} \cdot \text{m}}{(49 \text{ N})(0.55 \text{ m})} \quad \text{or} \quad \theta = \sin^{-1} \left[\frac{15 \text{ N} \cdot \text{m}}{(49 \text{ N})(0.55 \text{ m})} \right] = \boxed{34^\circ}$$

WebAssign Problem 2: One end of a meter stick is pinned to a table, so the stick can rotate freely in a plane parallel to the tabletop. Two forces, both parallel to the tabletop, are applied to the stick in such a way that the net torque is zero. One force has a magnitude of 2.00 N and is applied perpendicular to the length of the stick at the free end. The other force has a magnitude of 6.00 N and acts at a 30.0° angle with respect to the length of the stick. Where along the stick is the 6.00-N force applied? Express this distance with respect to the end that is pinned.

REASONING Since the meter stick does not move, it is in equilibrium. The forces and the torques, therefore, each add to zero. We can determine the location of the 6.00 N force, by using the condition that the sum of the torques must add to zero.



SOLUTION If we take counterclockwise torques as positive, then the torque of the first force about the pin is

$$\tau_1 = F_1 \ell_1 = (2.00 \text{ N})(1.00 \text{ m}) = 2.00 \text{ N} \cdot \text{m}$$

The torque due to the second force is

$$\tau_2 = -F_2 (\sin 30.0^\circ) \ell_2 = -(6.00 \text{ N})(\sin 30.0^\circ) \ell_2 = -(3.00 \text{ N}) \ell_2$$

The torque τ_2 is negative because the force F_2 tends to produce a clockwise rotation about the pinned end. Since the net torque is zero, we have

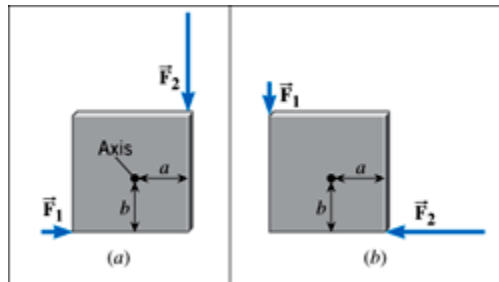
$$2.00 \text{ N} \cdot \text{m} + [-(3.00 \text{ N}) \ell_2] = 0$$

Thus,

$$\ell_2 = \frac{2.00 \text{ N} \cdot \text{m}}{3.00 \text{ N}} = \boxed{0.667 \text{ m}}$$

WebAssign Problem 3: A rotational axis is directed perpendicular to the plane of a square and is located as shown in the drawing. Two forces, \vec{F}_1 and \vec{F}_2 , are applied to diagonally opposite corners, and act along the sides of the square, first as shown in part *a*

and then as shown in part *b* of the drawing. In each case the net torque produced by the forces is zero. The square is one meter on a side, and the magnitude of \vec{F}_2 is three times that of \vec{F}_1 . Find the distances *a* and *b* that locate the axis.



REASONING AND SOLUTION The net torque about the axis in text drawing (a) is

$$\Sigma\tau = \tau_1 + \tau_2 = F_1 b - F_2 a = 0$$

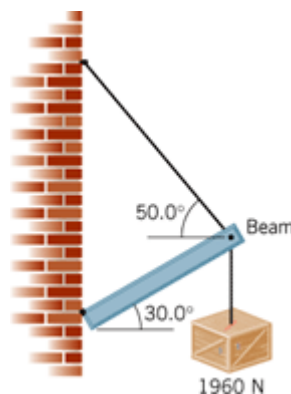
Considering that $F_2 = 3F_1$, we have $b - 3a = 0$. The net torque in drawing (b) is then

$$\Sigma\tau = F_1(1.00 \text{ m} - a) - F_2 b = 0 \quad \text{or} \quad 1.00 \text{ m} - a - 3b = 0$$

Solving the first equation for *b*, substituting into the second equation and rearranging, gives

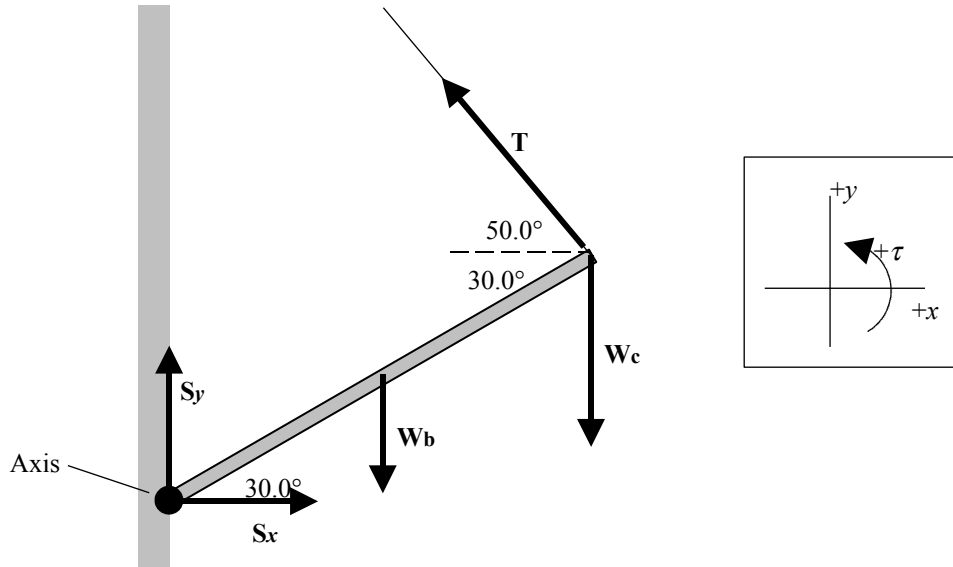
$$a = \boxed{0.100 \text{ m}} \quad \text{and} \quad b = \boxed{0.300 \text{ m}}$$

WebAssign Problem 4: A 1220-N uniform beam is attached to a vertical wall at one end and is supported by a cable at the other end. A 1960-N crate hangs from the far end of the beam. Using the data shown in the drawing, find (a) the magnitude of the tension in the wire and (b) the magnitude of the horizontal and vertical components of the force that the wall exerts on the left end of the beam.



REASONING The drawing shows the beam and the five forces that act on it: the horizontal and vertical components S_x and S_y that the wall exerts on the left end of the beam, the weight W_b of the beam, the force due to the weight W_c of the crate, and the tension T in the cable. The beam is uniform, so its center of gravity is at the

center of the beam, which is where its weight can be assumed to act. Since the beam is in equilibrium, the sum of the torques about any axis of rotation must be zero ($\Sigma \tau = 0$), and the sum of the forces in the horizontal and vertical directions must be zero ($\Sigma F_x = 0, \Sigma F_y = 0$). These three conditions will allow us to determine the magnitudes of S_x , S_y , and T .



SOLUTION

a. We will begin by taking the axis of rotation to be at the left end of the beam. Then the torques produced by S_x and S_y are zero, since their lever arms are zero. When we set the sum of the torques equal to zero, the resulting equation will have only one unknown, T , in it. Setting the sum of the torques produced by the three forces equal to zero gives (with L equal to the length of the beam)

$$\Sigma \tau = -W_b \left(\frac{1}{2} L \cos 30.0^\circ \right) - W_c \left(L \cos 30.0^\circ \right) + T \left(L \sin 80.0^\circ \right) = 0$$

Algebraically eliminating L from this equation and solving for T gives

$$T = \frac{W_b \left(\frac{1}{2} \cos 30.0^\circ \right) + W_c \left(\cos 30.0^\circ \right)}{\sin 80.0^\circ}$$

$$= \frac{(1220 \text{ N}) \left(\frac{1}{2} \cos 30.0^\circ \right) + (1960 \text{ N}) \left(\cos 30.0^\circ \right)}{\sin 80.0^\circ} = \boxed{2260 \text{ N}}$$

b. Since the beam is in equilibrium, the sum of the forces in the vertical direction must be zero:

$$\Sigma F_y = +S_y - W_b - W_c + T \sin 50.0^\circ = 0$$

Solving for S_y gives

$$S_y = W_b + W_c - T \sin 50.0^\circ = 1220 \text{ N} + 1960 \text{ N} - (2260 \text{ N}) \sin 50.0^\circ = \boxed{1450 \text{ N}}$$

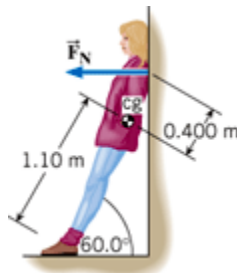
The sum of the forces in the horizontal direction must also be zero:

$$\Sigma F_x = +S_x - T \cos 50.0^\circ = 0$$

so that

$$S_x = T \cos 50.0^\circ = (2260 \text{ N}) \cos 50.0^\circ = \boxed{1450 \text{ N}}$$

WebAssign Problem 5: A woman who weighs $5.00 \times 10^2 \text{ N}$ is leaning against a smooth vertical wall, as the drawing shows. Find (a) the force \vec{F}_N (directed perpendicular to the wall) exerted on her shoulders by the wall and the (b) horizontal and (c) vertical components of the force exerted on her shoes by the ground.



REASONING AND SOLUTION

a. The net torque about an axis through the point of contact between the floor and her shoes is

$$\Sigma \tau = - (5.00 \times 10^2 \text{ N})(1.10 \text{ m}) \sin 30.0^\circ + F_N (\cos 30.0^\circ)(1.50 \text{ m}) = 0$$

$$F_N = \boxed{212 \text{ N}}$$

b. Newton's second law applied in the horizontal direction gives $F_h - F_N = 0$, so

$$F_h = \boxed{212 \text{ N}}.$$

c. Newton's second law in the vertical direction gives $F_v - W = 0$, so

$$F_v = \boxed{5.00 \times 10^2 \text{ N}}.$$

WebAssign Problem 6: A solid circular disk has a mass of 1.2 kg and a radius of 0.16 m. Each of three identical thin rods has a mass of 0.15 kg. The rods are attached perpendicularly to the plane of the disk at its outer edge to form a threelegged stool (see the drawing). Find the moment of inertia of the stool with respect to an axis that is perpendicular to the plane of the disk at its center. (*Hint: When considering the moment*

of inertia of each rod, note that all of the mass of each rod is located at the same perpendicular distance from the axis.)

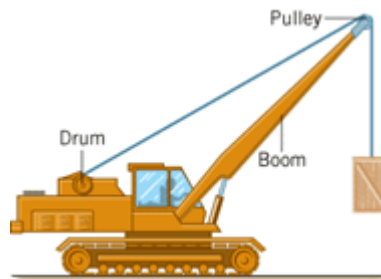


REASONING The moment of inertia of the stool is the sum of the individual moments of inertia of its parts. According to Table 9.1, a circular disk of radius R has a moment of inertia of $I_{\text{disk}} = \frac{1}{2} M_{\text{disk}} R^2$ with respect to an axis perpendicular to the disk center. Each thin rod is attached perpendicular to the disk at its outer edge. Therefore, each particle in a rod is located at a perpendicular distance from the axis that is equal to the radius of the disk. This means that each of the rods has a moment of inertia of $I_{\text{rod}} = M_{\text{rod}} R^2$.

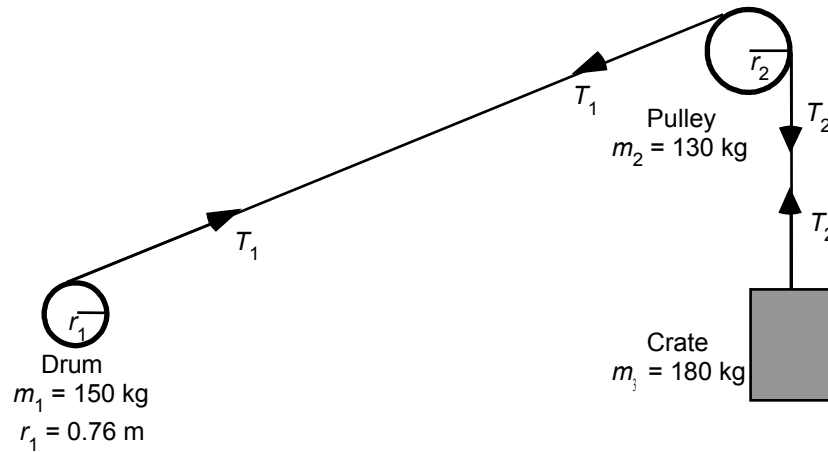
SOLUTION Remembering that the stool has three legs, we find that its moment of inertia is

$$I_{\text{stool}} = I_{\text{disk}} + 3I_{\text{rod}} = \frac{1}{2} M_{\text{disk}} R^2 + 3 M_{\text{rod}} R^2$$
$$= \frac{1}{2} (1.2 \text{ kg})(0.16 \text{ m})^2 + 3(0.15 \text{ kg})(0.16 \text{ m})^2 = \boxed{0.027 \text{ kg} \cdot \text{m}^2}$$

WebAssign Problem 7: See Multiple-Concept [Example 12](#) to review some of the concepts that come into play here. The crane shown in the drawing is lifting a 180-kg crate upward with an acceleration of 1.2 m/s^2 . The cable from the crate passes over a solid cylindrical pulley at the top of the boom. The pulley has a mass of 130 kg. The cable is then wound onto a hollow cylindrical drum that is mounted on the deck of the crane. The mass of the drum is 150 kg, and its radius is 0.76 m. The engine applies a counterclockwise torque to the drum in order to wind up the cable. What is the magnitude of this torque? Ignore the mass of the cable.



REASONING The drawing shows the drum, pulley, and the crate, as well as the tensions in the cord



Let T_1 represent the magnitude of the tension in the cord between the drum and the pulley. Then, the net torque exerted on the drum must be, according to Equation 9.7, $\Sigma\tau = I_1\alpha_1$, where I_1 is the moment of inertia of the drum, and α_1 is its angular acceleration. If we assume that the cable does not slip, then Equation 9.7 can be written as

$$\underbrace{-T_1 r_1 + \tau}_{\Sigma \tau} = \underbrace{(m_1 r_1^2)}_{I_1} \underbrace{\left(\frac{a}{r_1}\right)}_{\alpha_1} \quad (1)$$

where τ is the counterclockwise torque provided by the motor, and a is the acceleration of the cord ($a = 1.2 \text{ m/s}^2$). This equation cannot be solved for τ directly, because the tension T_1 is not known.

We next apply Newton's second law for rotational motion to the pulley in the drawing:

$$\underbrace{+T_1 r_2 - T_2 r_2}_{\Sigma \tau} = \underbrace{\left(\frac{1}{2} m_2 r_2^2\right)}_{I_2} \underbrace{\left(\frac{a}{r_2}\right)}_{\alpha_2} \quad (2)$$

where T_2 is the magnitude of the tension in the cord between the pulley and the crate, and I_2 is the moment of inertia of the pulley.

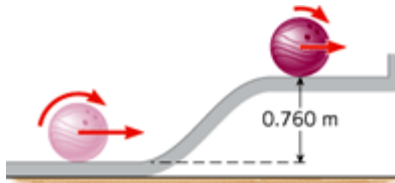
Finally, Newton's second law for translational motion ($\Sigma F_y = m a$) is applied to the crate, yielding

$$\underbrace{+ T_2 - m_3 g}_{\Sigma F_y} = m_3 a \quad (3)$$

SOLUTION Solving Equation (1) for T_1 and substituting the result into Equation (2), then solving Equation (2) for T_2 and substituting the result into Equation (3), results in the following value for the torque

$$\begin{aligned} \tau &= r_1 \left[a \left(m_1 + \frac{1}{2} m_2 + m_3 \right) + m_3 g \right] \\ &= (0.76 \text{ m}) \left[(1.2 \text{ m/s}^2) \left(150 \text{ kg} + \frac{1}{2} 130 \text{ kg} + 180 \text{ kg} \right) + (180 \text{ kg})(9.80 \text{ m/s}^2) \right] = \boxed{1700 \text{ N} \cdot \text{m}} \end{aligned}$$

WebAssign Problem 8: A cylindrically shaped space station is rotating about the axis of the cylinder to create artificial gravity. The radius of the cylinder is 82.5 m. The moment of inertia of the station without people is $3.00 \times 10^9 \text{ kg} \cdot \text{m}^2$. Suppose 500 people, with an average mass of 70.0 kg each, live on this station. As they move radially from the outer surface of the cylinder toward the axis, the angular speed of the station changes. What is the maximum possible percentage change in the station's angular speed due to the radial movement of the people?



REASONING AND SOLUTION The conservation of energy gives

$$mgh + (1/2) mv^2 + (1/2) I\omega^2 = (1/2) mv_0^2 + (1/2) I\omega_0^2$$

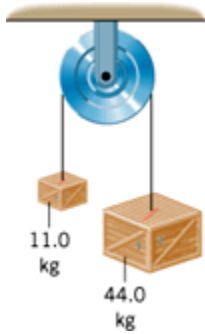
If the ball rolls without slipping, $\omega = v/R$ and $\omega_0 = v_0/R$. We also know $I = (2/5) mR^2$.

Substitution of the last two equations into the first and rearrangement gives

$$v = \sqrt{v_0^2 - \frac{10}{7} gh} = \sqrt{(3.50 \text{ m/s})^2 - \frac{10}{7} (9.80 \text{ m/s}^2) (0.760 \text{ m})} = \boxed{1.3 \text{ m/s}}$$

WebAssign Problem 9: Multiple-Concept [Example 12](#) deals with a situation that has similarities to this one and uses some of the same concepts that are needed here. See **Concept Simulation 9.4** at www.wiley.com/college/cutnell to review the principles involved in this problem. By means of a rope whose mass is negligible, two blocks are suspended over a pulley, as the drawing shows. The pulley can be treated as a uniform solid cylindrical disk. The downward acceleration of the 44.0-kg block is observed to be

exactly one-half the acceleration due to gravity. Noting that the tension in the rope is not the same on each side of the pulley, find the mass of the pulley.



REASONING AND SOLUTION Newton's law applied to the 11.0-kg object gives

$$T_2 - (11.0 \text{ kg})(9.80 \text{ m/s}^2) = (11.0 \text{ kg})(4.90 \text{ m/s}^2) \quad \text{or} \quad T_2 = 162 \text{ N}$$

A similar treatment for the 44.0-kg object yields

$$T_1 - (44.0 \text{ kg})(9.80 \text{ m/s}^2) = (44.0 \text{ kg})(-4.90 \text{ m/s}^2) \quad \text{or} \quad T_1 = 216 \text{ N}$$

For an axis about the center of the pulley

$$T_2 r - T_1 r = I(-\alpha) = (1/2) M r^2 (-a/r)$$

Solving for the mass M we obtain

$$M = (-2/a)(T_2 - T_1) = [-2/(4.90 \text{ m/s}^2)](162 \text{ N} - 216 \text{ N}) =$$

Practice conceptual problems:

Note: Chapter 9 problems were included in last week's solutions.

Chapter 10:

1. Two people pull on a horizontal spring that is attached to an immovable wall. Then, they detach it from the wall and pull on opposite ends of the horizontal spring. They pull just as hard in each case. In which situation, if either, does the spring stretch more? Account for your answer.

REASONING AND SOLUTION A horizontal spring is attached to an immovable wall.

Two people pull on the spring. They then detach it from the wall and pull on opposite ends of the horizontal spring. They pull just as hard in each case.

Suppose that each person pulls with a force \mathbf{P} . When the spring is attached to the wall, both people pull on the same end, so that they pull with a total force $2\mathbf{P}$. The wall exerts a force $-2\mathbf{P}$ at the other end of the spring. The tension in the spring has magnitude $2P$. When the two people pull on opposite ends of the horizontal spring, one person exerts a force \mathbf{P} at one end of the spring, and the other person exerts a force $-\mathbf{P}$ at the other end of the spring. The tension in the spring has magnitude P .

The tension in the spring is greater when it is attached to the immovable wall; therefore, the spring will stretch more in this case.

2. The drawing shows identical springs that are attached to a box in two different ways. Initially, the springs are unstrained. The box is then pulled to the right and released. In each case the initial displacement of the box is the same. At the moment of release, which box, if either, experiences the greater net force due to the springs? Provide a reason for your answer.



REASONING AND SOLUTION Identical springs are attached to a box in two different ways, as shown in the drawing in the text. Initially, the springs are unstrained. The box is then pulled to the right and released. The displacement of the box is the same in both cases.

For the box on the left, each spring is stretched and its displacement is $+\Delta x$, where the plus sign indicates a displacement to the right. According to Equation 10.2, the restoring force exerted by each spring on the box is $F_x = -k(+\Delta x)$. Thus, the net force due to both springs is $\Sigma F_x = -2k(+\Delta x)$.

For the box on the right, one spring is stretched and the other is compressed. However, the displacement of each spring is still $+\Delta x$. The net force exerted on the box is $\Sigma F_x = -k(+\Delta x) - k(+\Delta x) = -2k(+\Delta x)$.

Thus, both boxes experience the same net force.

4. A steel ball is dropped onto a concrete floor. Over and over again, it rebounds to its original height. Is the motion simple harmonic motion? Justify your answer.

REASONING AND SOLUTION Simple harmonic motion is the oscillatory motion that occurs when a restoring force of the form of Equation 10.2, $F_x = -kx$, acts on an object. The force changes continually as the displacement x changes.

A steel ball is dropped onto a concrete floor. Over and over again, it rebounds to its original height. During the time when the ball is in the air, either falling down or rebounding up, the only force acting on the ball is its weight, which is nearly constant, to the extent that the ball remains near the earth's surface. Thus, the motion of the bouncing ball is not simple harmonic motion.

9. Is more elastic potential energy stored in a spring when the spring is compressed by one centimeter than when it is stretched by the same amount? Explain.

REASONING AND SOLUTION The elastic potential energy that a spring has by virtue of being stretched or compressed is given by Equation 10.13: $PE_{\text{elastic}} = (1/2)kx^2$, where x is the amount by which the spring is stretched or compressed relative to its unstrained length. The amount of stretch or compression appears squared, so that the elastic potential energy is positive and independent of the sign of x . Therefore, the amount of elastic potential energy stored in a spring when it is compressed by one centimeter is the same as when it is stretched by the same amount.