HW9 Solutions

Ladybugs on a Rotating Disk

Part A. Answer: the same as the angular speed of ladybug 2

Part B. Answer: 2

Hint . Relation between linear and angular speeds
The relation between the linear speed v and angular speed ω of an object is given by

$$V = r\omega,$$

where is the distance between the object and the axis of rotation.

Part C. Answer: $\frac{a_2}{a_1} = 2$

Hint. Radial (centripetal) acceleration of an object moving on a circle

The magnitude of the radial (centripetal) acceleration of an object moving on a circle is called the centripetal acceleration. It is given by

$$a_c = r\omega^2 = \frac{V_t^2}{r},$$

where is the angular velocity of the object, is its tangential velocity, and is the distance from the axis of rotation.

Part D. Answer: +z

Part E. Answer: -y

9.4.IDENTIFY: $\alpha_z = d\omega_z/dt$. $\alpha_{\text{av-z}} = \dfrac{\Delta\omega_z}{\Delta t}$.

SET UP: $\dfrac{d}{dt}(t^2) = 2t$

EXECUTE: **(a)** $\alpha_z(t) = \dfrac{d\omega_z}{dt} = -2\beta t = (-1.60\ \text{rad/s}^3)t$.

(b) $\alpha_z(3.0\ \text{s}) = (-1.60\ \text{rad/s}^3)(3.0\ \text{s}) = -4.80\ \text{rad/s}^2$.

$$\alpha_{\text{av-z}} = \frac{\omega_z(3.0\ \text{s}) - \omega_z(0)}{3.0\ \text{s}} = \frac{-2.20\ \text{rad/s} - 5.00\ \text{rad/s}}{3.0\ \text{s}} = -2.40\ \text{rad/s}^2,$$

which is half as large (in magnitude) as the acceleration at $t = 3.0$ s.

EVALUATE: $\alpha_z(t)$ increases linearly with time, so $\alpha_{\text{av-}z} = \dfrac{\alpha_z(0)+\alpha_z(3.0\text{ s})}{2}$. $\alpha_z(0)=0.$

9.15. IDENTIFY: Apply constant angular acceleration equations.
SET UP: Let the direction the flywheel is rotating be positive.
$\theta-\theta_0 = 200\text{ rev}, \omega_{0z} = 500\text{ rev/min} = 8.333\text{ rev/s}, t = 30.0\text{ s}.$

EXECUTE: **(a)** $\theta-\theta_0 = \left(\dfrac{\omega_{0z}+\omega_z}{2}\right)t$ gives $\omega_z = 5.00\text{ rev/s} = 300\text{ rpm}$

(b) Use the information in part (a) to find α_z: $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = -0.1111\text{ rev/s}^2$. Then $\omega_z = 0$, $\alpha_z = -0.1111\text{ rev/s}^2, \omega_{0z} = 8.333\text{ rev/s}$ in $\omega_z = \omega_{0z}+\alpha_z t$ gives $t = 75.0\text{ s}$ and $\theta-\theta_0 = \left(\dfrac{\omega_{0z}+\omega_z}{2}\right)t$ gives $\theta-\theta_0 = 312\text{ rev.}$

EVALUATE: The mass and diameter of the flywheel are not used in the calculation.

9.24. IDENTIFY: Apply constant angular acceleration equations. $v = r\omega.$ A point on the rim has both tangential and radial components of acceleration.
SET UP: $a_{\tan} = r\alpha$ and $a_{\text{rad}} = r\omega^2.$

EXECUTE: **(a)** $\omega_z = \omega_{0z} + \alpha_z t \;=\; 0.250\text{ rev/s} + (0.900\text{ rev/s}^2)(0.200\text{ s}) = 0.430\text{ rev/s}$
(Note that since ω_{0z} and α_z are given in terms of revolutions, it's not necessary to convert to radians).

(b) $\omega_{\text{av-z}}\Delta t = (0.340\text{ rev/s})(0.2\text{ s}) = 0.068\text{ rev.}$

(c) Here, the conversion to radians must be made to use $v = r\omega$, and

$$v = r\omega = \left(\frac{0.750\text{ m}}{2}\right)(0.430\text{ rev/s})(2\pi\text{ rad/rev}) = 1.01\text{ m/s.}$$

(d) Combining $a_{\text{rad}} = r\omega^2$ and $a_{\tan} = R\alpha$, $a = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2}.$

$a = \sqrt{\left[((0.430\text{ rev/s})(2\pi\text{ rad/rev}))^2(0.375\text{ m})\right]^2 + \left[(0.900\text{ rev/s}^2)(2\pi\text{ rad/rev})(0.375\text{ m})\right]^2}.$

$a = 3.46\text{ m/s}^2.$

EVALUATE: If the angular acceleration is constant, $a_{\tan}$ is constant but a_{rad} increases as ω increases.

9.30. IDENTIFY: Treat each block as a point mass, so for each block $I = mr^2$, where *r* is the distance of the block from the axis. The total *I* for the object is the sum of the *I* for each of its pieces.

SET UP: In part (a) two blocks are a distance $L/2$ from the axis and the third block is on the axis. In
part (b) two blocks are a distance $L/4$ from the axis and one is a distance $3L/4$ from the axis.

EXECUTE: (a) $I = 2m(L/2)^2 = \frac{1}{2}mL^2.$

(b) $I = 2m(L/4)^2 + m(3L/4)^2 = \dfrac{1}{16}mL^2(2+9) = \dfrac{11}{16}mL^2.$

EVALUATE: For the same object *I* is in general different for different axes.

9.34. IDENTIFY: $K = \frac{1}{2}I\omega^2.$ Use Table 9.2 to calculate *I*.

SET UP: $I = \frac{1}{12}ML^2.$ $1\ \text{rpm} = 0.1047\ \text{rad/s}$

EXECUTE: (a) $I = \frac{1}{12}(117\ \text{kg})(2.08\ \text{m})^2 = 42.2\ \text{kg}\cdot\text{m}^2.$

$\omega = (2400\ \text{rev/min})\left(\dfrac{0.1047\ \text{rad/s}}{1\ \text{rev/min}}\right) = 251\ \text{rad/s}.$ $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(42.2\ \text{kg}\cdot\text{m}^2)(251\ \text{rad/s})^2 = 1.33\times10^6\ \text{J}.$

(b) $K_1 = \frac{1}{12}M_1L_1^2\omega_1^2,$ $K_2 = \frac{1}{12}M_2L_2^2\omega_2^2.$ $L_1 = L_2$ and $K_1 = K_2,$ so $M_1\omega_1^2 = M_2\omega_2^2.$

$\omega_2 = \omega_1\sqrt{\dfrac{M_1}{M_2}} = (2400\ \text{rpm})\sqrt{\dfrac{M_1}{0.750M_1}} = 2770\ \text{rpm}$

EVALUATE: The rotational kinetic energy is proportional to the square of the angular speed and directly proportional to the mass of the object.

9.45. IDENTIFY: With constant acceleration, we can use kinematics to find the speed of the falling object. Then we can apply the work-energy expression to the entire system and find the moment of inertia of the wheel. Finally, using its radius we can find its mass, the target variable.

SET UP: With constant acceleration, $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t.$ The angular velocity of the wheel is related to the linear velocity of the falling mass by $\omega_z = \dfrac{v_y}{R}.$ The work-energy theorem is $K_1 + U_1 + W_\text{other} = K_2 + U_2,$ and the moment of inertia of a uniform disk is $I = \frac{1}{2}MR^2.$

EXECUTE: Find $v_y,$ the velocity of the block after it has descended 3.00 m.
$y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $v_y = \dfrac{2(y - y_0)}{t} = \dfrac{2(3.00\ \text{m})}{2.00\ \text{s}} = 3.00\ \text{m/s}.$ For the wheel,
$\omega_z = \dfrac{v_y}{R} = \dfrac{3.00\ \text{m/s}}{0.280\ \text{m}} = 10.71\ \text{rad/s}.$ Apply the work-energy expression:
$K_1 + U_1 + W_\text{other} = K_2 + U_2,$ giving $mg(3.00\ \text{m}) = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2.$ Solving for *I* gives

$I = \frac{2}{\omega^2}\left[mg(3.00\text{ m}) - \frac{1}{2}mv^2\right]$. $I = \frac{2}{(10.71\text{ rad/s})^2}\left[(4.20\text{ kg})(9.8\text{ m/s}^2)(3.00\text{ m}) - \frac{1}{2}(4.20\text{ kg})(3.00\text{ m/s})^2\right]$.

$I = 1.824\text{ kg}\cdot\text{m}^2$. For a solid disk, $I = \frac{1}{2}MR^2$ gives $M = \frac{2I}{R^2} = \frac{2(1.824\text{ kg}\cdot\text{m}^2)}{(0.280\text{ m})^2} = 46.5\text{ kg}$.

EVALUATE: The gravitational potential of the falling object is converted into the kinetic energy of that object and the rotational kinetic energy of the wheel.

9.52. **IDENTIFY:** Use the equations in Table 9.2. *I* for the rod is the sum of *I* for each segment. The parallel-axis theorem says $I_\text{p} = I_\text{cm} + Md^2$.

SET UP: The bent rod and axes *a* and *b* are shown in Figure 9.52. Each segment has length $L/2$ and mass $M/2$.

EXECUTE: **(a)** For each segment the moment of inertia is for a rod with mass $M/2$, length $L/2$ and the axis through one end. For one segment, $I_\text{s} = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{24}ML^2$. For the rod, $I_\text{a} = 2I_\text{s} = \frac{1}{12}ML^2$.

(b) The center of mass of each segment is at the center of the segment, a distance of $L/4$ from each end. For each segment, $I_\text{cm} = \frac{1}{12}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{96}ML^2$. Axis *b* is a distance $L/4$ from the cm of each segment, so for each segment the parallel axis theorem gives *I* for axis *b* to be $I_\text{s} = \frac{1}{96}ML^2 + \frac{M}{2}\left(\frac{L}{4}\right)^2 = \frac{1}{24}ML^2$ and $I_\text{b} = 2I_\text{s} = \frac{1}{12}ML^2$.

EVALUATE: *I* for these two axes are the same.

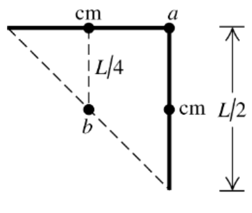

Figure 9.52

9.55. **IDENTIFY:** Apply $I = \int r^2\,dm$ and $M = \int dm$.

SET UP: For this case, $dm = \gamma x\,dx$.

EXECUTE: **(a)** $M = \int dm = \int_0^L \gamma x\, dx = \gamma \frac{x^2}{2}\Big|_0^L = \frac{\gamma L^2}{2}$

(b) $I = \int_0^L x^2(\gamma x)dx = \gamma \frac{x^4}{4}\Big|_0^L = \frac{\gamma L^4}{4} = \frac{M}{2}L^2.$ This is larger than the moment of inertia of a uniform rod of the same mass and length, since the mass density is greater farther away from the axis than nearer the axis.

(c) $I = \int_0^L (L-x)^2 \gamma x dx = \gamma \int_0^L (L^2 x - 2Lx^2 + x^3)dx = \gamma \left(L^2 \frac{x^2}{2} - 2L\frac{x^3}{3} + \frac{x^4}{4} \right)\Big|_0^L = \gamma \frac{L^4}{12} = \frac{M}{6}L^2.$

This is a third of the result of part (b), reflecting the fact that more of the mass is concentrated at the
right end.

EVALUATE: For a uniform rod with an axis at one end, $I = \frac{1}{3}ML^2.$ The result in (b) is larger than this and the result in (c) is smaller than this.

9.75. **IDENTIFY:** Apply conservation of energy to the system consisting of blocks *A* and *B* and the pulley.

SET UP: The system at points 1 and 2 of its motion is sketched in Figure 9.75.

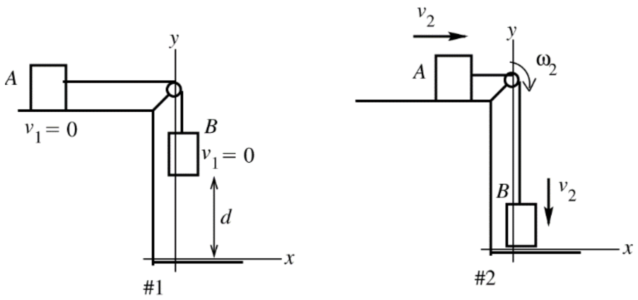

Figure 9.75

Use the work-energy relation $K_1 + U_1 + W_{\text{other}} = K_2 + U_2.$ Use coordinates where $+y$ is upward and where the origin is at the position of block *B* after it has descended. The tension in the rope does positive work on block *A* and negative work of the same magnitude on block *B*, so the net work done by the tension in the rope is zero. Both blocks have the same speed.

EXECUTE: Gravity does work on block *B* and kinetic friction does work on block *A*. Therefore $W_{\text{other}} = W_f = -\mu_k m_A g d.$

$K_1 = 0$ (system is released from rest)

$U_1 = m_B g y_{B1} = m_B g d;\;\; U_2 = m_B g y_{B2} = 0$

$K_2 = \frac{1}{2}m_A v_2^2 + \frac{1}{2}m_B v_2^2 + \frac{1}{2}I\omega_2^2.$

But $v(\text{blocks}) = R\omega(\text{pulley}),$ so $\omega_2 = v_2/R$ and

$K_2 = \frac{1}{2}(m_A + m_B)v_2^2 + \frac{1}{2}I(v_2/R)^2 = \frac{1}{2}(m_A + m_B + I/R^2)v_2^2$

Putting all this into the work-energy relation gives

$$m_B gd - \mu_k m_A gd = \tfrac{1}{2}(m_A + m_B + I/R^2)v_2^2$$

$$(m_A + m_B + I/R^2)v_2^2 = 2gd(m_B - \mu_k m_A)$$

$$v_2 = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + I/R^2}}$$

EVALUATE: If $m_B >> m_A$ and $I/R^2,$ then $v_2 = \sqrt{2gd};$ block *B* falls freely. If *I* is very large, v_2 is very small. Must have $m_B > \mu_k m_A$ for motion, so the weight of *B* will be larger than the friction force on *A*. I/R^2 has units of mass and is in a sense the "effective mass" of the pulley.

9.8. IDENTIFY: $\alpha_z = \frac{d\omega_z}{dt}.$ $\theta - \theta_0 = \omega_{\text{av-z}}t.$ When ω_z is linear in *t*, $\omega_{\text{av-z}}$ for the time interval t_1 to t_2 is $\omega_{\text{av-z}} = \frac{\omega_{z1} + \omega_{z2}}{t_2 - t_1}.$

SET UP: From the information given, $\alpha_z = \frac{\Delta\omega}{\Delta t} = \frac{4.00\text{ rad/s} - (-6.00\text{ rad/s})}{7.00\text{ s}} = 1.429\text{ rad/s}^2.$

$\omega_z(t) = -6.00\text{ rad/s} + (1.429\text{ rad/s}^2)t.$

EXECUTE: **(a)** The angular acceleration is positive, since the angular velocity increases steadily from a negative value to a positive value.

(b) It takes time $t = -\frac{\omega_{0z}}{\alpha_z}$ = –(–6.00 rad/s)/(1.429 rad/s^2) = 4.20 s for the wheel to stop $(\omega_z = 0).$ During this time its speed is decreasing. For the next 2.80 s its speed is increasing from $0\text{ rad/s to } +4.00\text{ rad/s}.$

(c) The average angular velocity is $\frac{-6.00\text{ rad/s} + 4.00\text{ rad/s}}{2} = -1.00\text{ rad/s}.$ $\theta - \theta_0 = \omega_{\text{av-z}}t$ then leads to displacement of –7.00 rad after 7.00 s.

EVALUATE: When α_z and ω_z have the same sign, the angular speed is increasing; this is the case for $t = 4.20\text{ s}$ to $t = 7.00\text{ s}.$ When α_z and ω_z have opposite signs, the angular speed is decreasing; this is the case between $t = 0$ and $t = 4.20\text{ s}.$