HW10 Solutions

10.3. **IDENTIFY** and **SET UP:** Use $\tau = Fl$ to calculate the magnitude of each torque and use the right-hand rule (Figure 10.4 in the textbook) to determine the direction. Consider Figure 10.3.

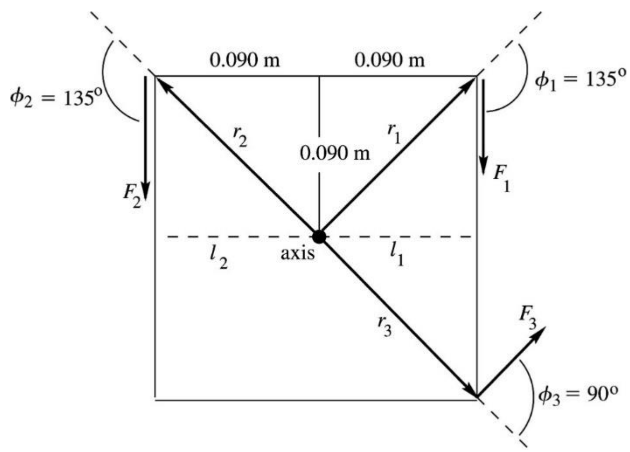

Figure 10.3

Let counterclockwise be the positive sense of rotation.

EXECUTE: $r_1 = r_2 = r_3 = \sqrt{(0.090\text{ m})^2 + (0.090\text{ m})^2} = 0.1273\text{ m}$

$\tau_1 = -F_1 l_1$

$l_1 = r_1 \sin\phi_1 = (0.1273\text{ m})\sin 135° = 0.0900\text{ m}$

$\tau_1 = -(18.0\text{ N})(0.0900\text{ m}) = -1.62\text{ N}\cdot\text{m}$

$\vec{\tau}_1$ is directed into paper

$\tau_2 = +F_2 l_2$

$l_2 = r_2 \sin\phi_2 = (0.1273\text{ m})\sin 135° = 0.0900\text{ m}$

$\tau_2 = +(26.0\text{ N})(0.0900\text{ m}) = +2.34\text{ N}\cdot\text{m}$

$\vec{\tau}_2$ is directed out of paper

$\tau_3 = +F_3 l_3$

$l_3 = r_3 \sin\phi_3 = (0.1273\text{ m})\sin 90° = 0.1273\text{ m}$

$\tau_3 = +(14.0\text{ N})(0.1273\text{ m}) = +1.78\text{ N}\cdot\text{m}$

$\vec{\tau}_3$ is directed out of paper

$\sum\tau = \tau_1 + \tau_2 + \tau_3 = -1.62\text{ N}\cdot\text{m} + 2.34\text{ N}\cdot\text{m} + 1.78\text{ N}\cdot\text{m} = 2.50\text{ N}\cdot\text{m}$

EVALUATE: The net torque is positive, which means it tends to produce a counterclockwise rotation; the vector torque is directed out of the plane of the paper. In summing the torques it is important to include $+$ or $-$ signs to show direction.

10.10. IDENTIFY: Apply $\sum\tau_z = I\alpha_z$ to the wheel. The acceleration *a* of a point on the cord and the angular acceleration α of the wheel are related by $a = R\alpha$.

SET UP: Let the direction of rotation of the wheel be positive. The wheel has the shape of a disk and $I = \frac{1}{2}MR^2$. The free-body diagram for the wheel is sketched in Figure 10.10a for a horizontal pull and in Figure 10.10b for a vertical pull. *P* is the pull on the cord and *F* is the force exerted on the wheel by the axle.

EXECUTE: **(a)** $\alpha_z = \frac{\tau_z}{I} = \frac{(40.0 \text{ N})(0.250 \text{ m})}{\frac{1}{2}(9.20 \text{ kg})(0.250 \text{ m})^2} = 34.8 \text{ rad/s}^2.$

$a = R\alpha = (0.250 \text{ m})(34.8 \text{ rad/s}^2) = 8.70 \text{ m/s}^2.$

(b) $F_x = -P,\ \ F_y = Mg.\ \ F = \sqrt{P^2 + (Mg)^2} = \sqrt{(40.0 \text{ N})^2 + ([9.20 \text{ kg}][9.80 \text{ m/s}^2])^2} = 98.6 \text{ N}.$

$\tan\phi = \frac{|F_y|}{|F_x|} = \frac{Mg}{P} = \frac{(9.20 \text{ kg})(9.80 \text{ m/s}^2)}{40.0 \text{ N}}$ and $\phi = 66.1°.$ The force exerted by the axle has magnitude 98.6 N and is directed at $66.1°$ above the horizontal, away from the direction of the pull on the cord.

(c) The pull exerts the same torque as in part (a), so the answers to part (a) don't change. In part (b), $F + P = Mg$ and $F = Mg - P = (9.20 \text{ kg})(9.80 \text{ m/s}^2) - 40.0 \text{ N} = 50.2 \text{ N}.$ The force exerted by the axle has magnitude 50.2 N and is upward.

EVALUATE: The weight of the wheel and the force exerted by the axle produce no torque because they act at the axle.

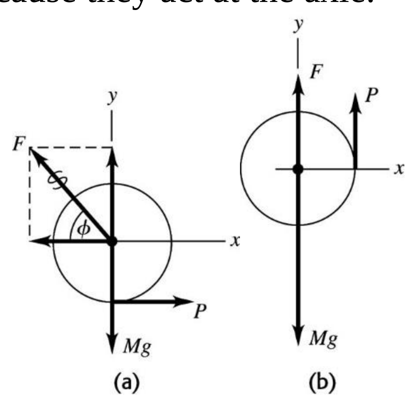

Figure 10.10

10.16. IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to each box and $\sum\tau_z = I\alpha_z$ to the pulley. The magnitude *a* of the acceleration of each box is related to the magnitude of the angular acceleration α of the pulley by $a = R\alpha$.

SET UP: The free-body diagrams for each object are shown in Figure 10.16. For the pulley, $R = 0.250 \text{ m}$ and $I = \frac{1}{2}MR^2$. T_1 and T_2 are the tensions in the wire on either side of the pulley. $m_1 = 12.0$ kg, $m_2 = 5.00$ kg and $M = 2.00$ kg. $\vec{F}$ is the force that the axle exerts on the pulley. For the pulley, let clockwise rotation be positive.

EXECUTE: **(a)** $\sum F_x = ma_x$ for the 12.0 kg box gives $T_1 = m_1 a$. $\sum F_y = ma_y$ for the 5.00 kg weight gives $m_2 g - T_2 = m_2 a$. $\sum \tau_z = I\alpha_z$ for the pulley gives $(T_2 - T_1)R = (\frac{1}{2}MR^2)\alpha$. $a = R\alpha$ and $T_2 - T_1 = \frac{1}{2}Ma$. Adding these three equations gives $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ and

$$a = \left(\frac{m_2}{m_1 + m_2 + \frac{1}{2}M}\right)g = \left(\frac{5.00\text{ kg}}{12.0\text{ kg} + 5.00\text{ kg} + 1.00\text{ kg}}\right)(9.80\text{ m/s}^2) = 2.72\text{ m/s}^2.$$ Then

$T_1 = m_1 a = (12.0\text{ kg})(2.72\text{ m/s}^2) = 32.6\text{ N}$. $m_2 g - T_2 = m_2 a$ gives

$T_2 = m_2(g - a) = (5.00\text{ kg})(9.80\text{ m/s}^2 - 2.72\text{ m/s}^2) = 35.4\text{ N}$. The tension to the left of the pulley is 32.6 N and below the pulley it is 35.4 N.

(b) $a = 2.72\text{ m/s}^2$

(c) For the pulley, $\sum F_x = ma_x$ gives $F_x = T_1 = 32.6$ N and $\sum F_y = ma_y$ gives

$F_y = Mg + T_2 = (2.00\text{ kg})(9.80\text{ m/s}^2) + 35.4\text{ N} = 55.0\text{ N}$.

EVALUATE: The equation $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ says that the external force $m_2 g$ must accelerate all three objects.

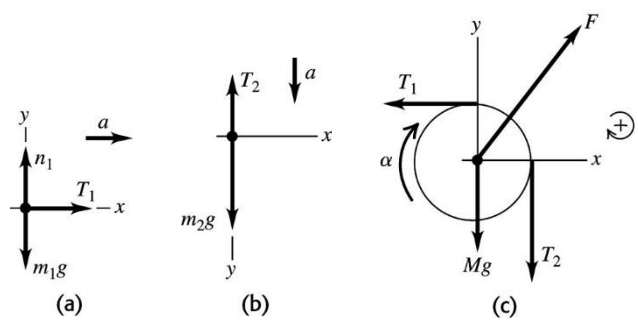

Figure 10.16

10.20. **IDENTIFY:** Only gravity does work, so $W_{\text{other}} = 0$ and conservation of energy gives $K_1 + U_1 = K_2 + U_2$. $K_2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$.

SET UP: Let $y_2 = 0$, so $U_2 = 0$ and $y_1 = 0.750$ m. The hoop is released from rest so $K_1 = 0$. $v_{\text{cm}} = R\omega$. For a hoop with an axis at its center, $I_{\text{cm}} = MR^2$.

EXECUTE: **(a)** Conservation of energy gives $U_1 = K_2$.

$K_2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}(MR^2)\omega^2 = MR^2\omega^2$, so $MR^2\omega^2 = Mgy_1$. $\omega = \frac{\sqrt{gy_1}}{R} = \frac{\sqrt{(9.80\text{ m/s}^2)(0.750\text{ m})}}{0.0800\text{ m}} = 33.9\text{ rad/s}$.

(b) $v = R\omega = (0.0800\text{ m})(33.9\text{ rad/s}) = 2.71\text{ m/s}$

EVALUATE: An object released from rest and falling in free fall for 0.750 m attains a speed of $\sqrt{2g(0.750\text{ m})} = 3.83\text{ m/s}$. The final speed of the hoop is less than this because some of its energy is in kinetic energy of rotation. Or, equivalently, the upward tension causes the magnitude of the net force of the hoop to be less than its weight.

10.36. **IDENTIFY:** $L = I\omega$ and $I = I_{\text{disk}} + I_{\text{woman}}$.

SET UP: $\omega = 0.80\ \text{rev/s} = 5.026\ \text{rad/s}$. $I_{\text{disk}} = \frac{1}{2}m_{\text{disk}}R^2$ and $I_{\text{woman}} = m_{\text{woman}}R^2$.

EXECUTE: $I = (55\ \text{kg} + 50.0\ \text{kg})(4.0\ \text{m})^2 = 1680\ \text{kg}\cdot\text{m}^2$.

$L = (1680\ \text{kg}\cdot\text{m}^2)(5.026\ \text{rad/s}) = 8.4\times10^3\ \text{kg}\cdot\text{m}^2/\text{s}$.

EVALUATE: The disk and the woman have similar values of *I*, even though the disk has twice the mass.

10.41. **IDENTIFY:** Apply conservation of angular momentum.

SET UP: For a uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$.

EXECUTE: The moment of inertia is proportional to the square of the radius, and so the angular velocity will be proportional to the inverse of the square of the radius, and the final angular velocity is

$$\omega_2 = \omega_1\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2\pi\ \text{rad}}{(30\ \text{d})(86{,}400\ \text{s/d})}\right)\left(\frac{7.0\times10^5\ \text{km}}{16\ \text{km}}\right)^2 = 4.6\times10^3\ \text{rad/s}.$$

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EVALUATE: $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega$. *L* is constant and ω increases by a large factor, so there is a large increase in the rotational kinetic energy of the star. This energy comes from potential energy associated with the gravity force within the star.

10.44. **IDENTIFY:** Apply conservation of angular momentum to the collision.

SET UP: Let the width of the door be *l*. The initial angular momentum of the mud is $mv(l/2)$, since it strikes the door at its center. For the axis at the hinge, $I_{\text{door}} = \frac{1}{3}Ml^2$ and $I_{\text{mud}} = m(l/2)^2$.

EXECUTE: $\omega = \dfrac{L}{I} = \dfrac{mv(l/2)}{(1/3)Ml^2 + m(l/2)^2}$.

$$\omega = \frac{(0.500\text{ kg})(12.0\text{ m/s})(0.500\text{ m})}{(1/3)(40.0\text{ kg})(1.00\text{ m})^2 + (0.500\text{ kg})(0.500\text{ m})^2} = 0.223\text{ rad/s}.$$

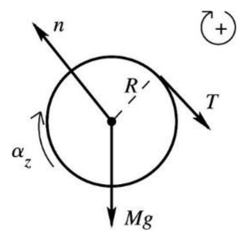
Ignoring the mass of the mud in the denominator of the above expression gives $\omega = 0.225\text{ rad/s}$,

so the mass of the mud in the moment of inertia does affect the third significant figure.

EVALUATE: Angular momentum is conserved but there is a large decrease in the kinetic energy of the system.

10.62. IDENTIFY: Apply $\sum\tau_z = I\alpha_z$ to the flywheel and $\sum\vec{F} = m\vec{a}$ to the block. The target variables are the tension in the string and the acceleration of the block.

(a) SET UP: Apply $\sum\tau_z = I\alpha_z$ to the rotation of the flywheel about the axis. The free-body diagram for the flywheel is given in Figure 10.62a.



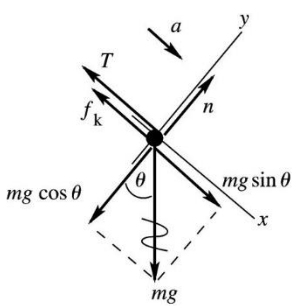
EXECUTE: The forces n and Mg act at the axis so have zero torque.

$\sum\tau_z = TR$

$TR = I\alpha_z$

Figure 10.62a

SET UP: Apply $\sum\vec{F} = m\vec{a}$ to the translational motion of the block. The free-body diagram for the block is given in Figure 10.62b.



EXECUTE: $\sum F_y = ma_y$

$n - mg\cos 36.9° = 0$

$n = mg\cos 36.9°$

$f_k = \mu_k n = \mu_k mg\cos 36.9°$

Figure 10.62b

$\sum F_x = ma_x$

$mg\sin 36.9° - T - \mu_k mg\cos 36.9° = ma$

$mg(\sin 36.9° - \mu_k \cos 36.9°) - T = ma$

But we also know that $a_{block} = R\alpha_{wheel}$, so $\alpha = a/R$. Using this in the $\sum\tau_z = I\alpha_z$ equation gives $TR = Ia/R$ and $T = (I/R^2)a$. Use this to replace T in the $\sum F_x = ma_x$ equation:

$$mg(\sin 36.9° - \mu_k \cos 36.9°) - (I/R^2)a = ma$$

$$a = \frac{mg(\sin 36.9° - \mu_k \cos 36.9°)}{m + I/R^2}$$

$$a = \frac{(5.00\text{ kg})(9.80\text{ m/s}^2)[\sin 36.9° - (0.25)\cos 36.9°]}{5.00\text{ kg} + 0.500\text{ kg}\cdot\text{m}^2/(0.200\text{ m})^2} = 1.12\text{ m/s}^2.$$

(b) $T = \dfrac{0.500\text{ kg}\cdot\text{m}^2}{(0.200\text{ m})^2}(1.12\text{ m/s}^2) = 14.0\text{ N}$

EVALUATE: If the string is cut the block will slide down the incline with $a = g\sin 36.9° - \mu_k g\cos 36.9° = 3.92\text{ m/s}^2$. The actual acceleration is less than this because $mg\sin 36.9°$ must also accelerate the flywheel. $mg\sin 36.9° - f_k = 19.6\text{ N}$. T is less than this; there must be more force on the block directed down the incline than up the incline since the block accelerates down the incline.

10.64. IDENTIFY: Apply both $\sum\vec{F} = m\vec{a}$ and $\sum\tau_z = I\alpha_z$ to the motion of the roller. Rolling without slipping means $a_{cm} = R\alpha$. Target variables are a_{cm} and f.

SET UP: The free-body diagram for the roller is given in Figure 10.64.

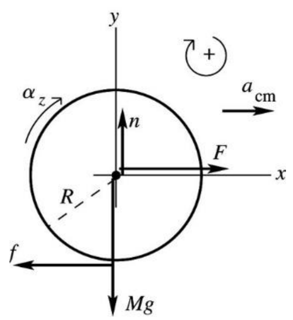


Figure 10.64

EXECUTE: Apply $\sum\vec{F} = m\vec{a}$ to the translational motion of the center of mass:

$\sum F_x = ma_x$

$F - f = Ma_{cm}$

Apply $\sum\tau_z = I\alpha_z$ to the rotation about the center of mass:

$\sum\tau_z = fR$

thin-walled hollow cylinder: $I = MR^2$

Then $\sum\tau_z = I\alpha_z$ implies $fR = MR^2\alpha$.

But $\alpha_{cm} = R\alpha$, so $f = Ma_{cm}$.

Using this in the $\sum F_x = ma_x$ equation gives $F - Ma_{cm} = Ma_{cm}$.

$a_{cm} = F/2M$, and then $f = Ma_{cm} = M(F/2M) = F/2$.

EVALUATE: If the surface were frictionless the object would slide without rolling and the acceleration would be $a_{cm} = F/M$. The acceleration is less when the object rolls.

Torque Magnitude Ranking Task

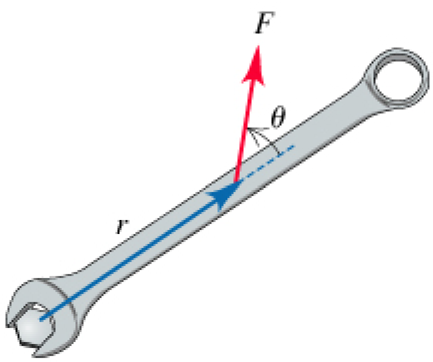
Part A

Hint 1. Definition of torque

Torque is a measure of the "twist" that an applied force exerts on an object. Mathematically, torque is defined as

$$\tau = rF\sin\theta,$$

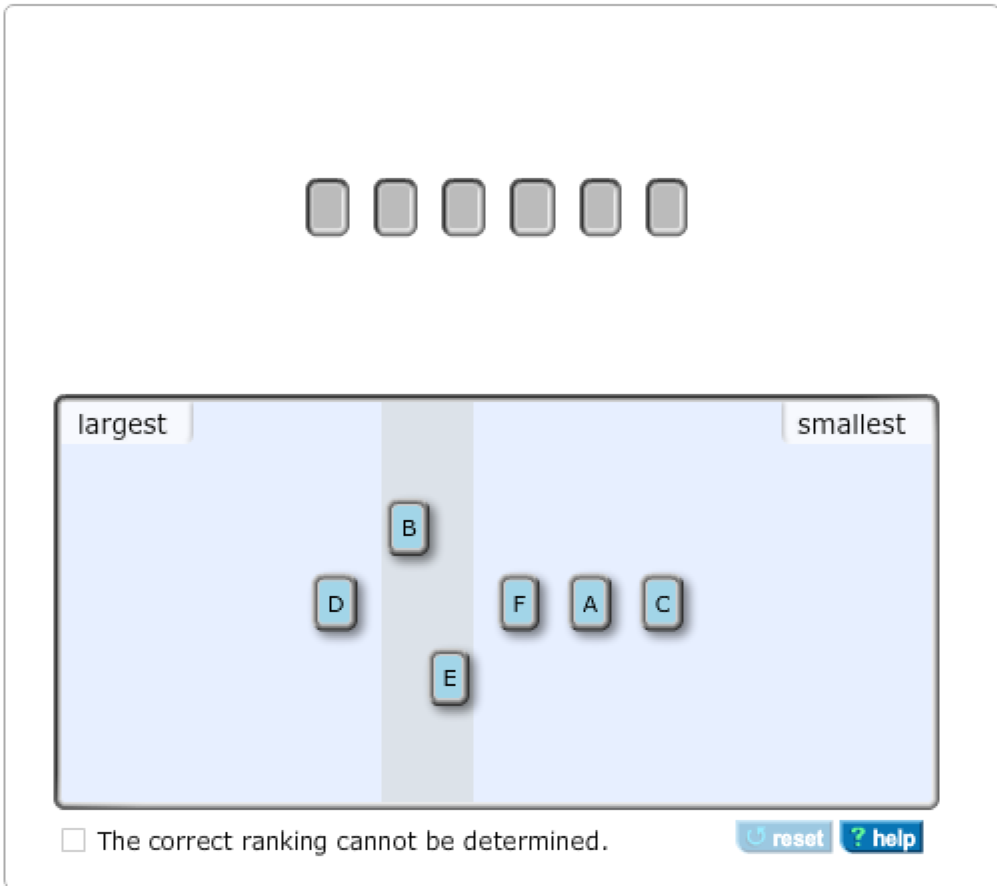
where r is the magnitude of the displacement vector from the rotation axis to the point of application of the force of magnitude F, and θ is the angle between this displacement and the applied force, as shown in the figure.



The direction of a torque can be either counterclockwise (as above) or clockwise. This is determined by the direction the object will rotate under the action of the force.

Hint 2. Maximum Torque

Based on the mathematical definition of torque, torque is maximized when the force is large in magnitude, located a large distance from the axis of interest, and oriented perpendicular to the displacement $\vec{r}$, which is often referred to as the lever arm of the force.



10.6. **Identify:** Knowing the force on a bar and the point where it acts, we want to find the position vector for the point where the force acts and the torque the force exerts on the bar.

Set Up: The position vector is $\vec{r} = x\hat{i} + y\hat{j}$ and the torque is $\vec{\tau} = \vec{r} \times \vec{F}$.

Execute: **(a)** Using $x = 3.00$ m and $y = 4.00$ m, we have $\vec{r} = (3.00\text{ m})\hat{i} + (4.00\text{ m})\hat{j}$.

(b) $\vec{\tau} = \vec{r} \times \vec{F} = [(3.00\text{ m})\hat{i} + (4.00\text{ m})\hat{j}] \times [(7.00\text{ N})\hat{i} + (-3.00\text{ N})\hat{j}]$.

$\vec{\tau} = (-9.00\text{ N}\cdot\text{m})\hat{k} + (-28.0\text{ N}\cdot\text{m})(-\hat{k}) = (-37.0\text{ N}\cdot\text{m})\hat{k}$. The torque has magnitude $37.0\text{ N}\cdot\text{m}$ and is in the $-z$-direction.

Evaluate: Applying the right-hand rule for the vector product to $\vec{r} \times \vec{F}$ shows that the torque must be in the $-z$-direction because it is perpendicular to both $\vec{r}$ and $\vec{F}$, which are both in the x-y plane.