HW1 Solution

1.1. **IDENTIFY:** Convert units from mi to km and from km to ft.

SET UP: 1 in. = 2.54 cm, 1 km = 1000 m, 12 in. = 1 ft, 1 mi = 5280 ft.

EXECUTE: **(a)** $1.00\text{ mi} = (1.00\text{ mi})\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)\left(\frac{2.54\text{ cm}}{1\text{ in.}}\right)\left(\frac{1\text{ m}}{10^2\text{ cm}}\right)\left(\frac{1\text{ km}}{10^3\text{ m}}\right) = 1.61\text{ km}$

(b) $1.00\text{ km} = (1.00\text{ km})\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{10^2\text{ cm}}{1\text{ m}}\right)\left(\frac{1\text{ in.}}{2.54\text{ cm}}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right) = 3.28\times10^3\text{ ft}$

EVALUATE: A mile is a greater distance than a kilometer. There are 5280 ft in a mile but only 3280 ft in a km.

1.14. **IDENTIFY:** When numbers are multiplied or divided, the number of significant figures in the result can be no greater than in the factor with the fewest significant figures. When we add or subtract numbers it is the location of the decimal that matters.

SET UP: 12 mm has two significant figures and 5.98 mm has three significant figures.

EXECUTE: **(a)** $(12\text{ mm})\times(5.98\text{ mm}) = 72\text{ mm}^2$ (two significant figures)

(b) $\frac{5.98\text{ mm}}{12\text{ mm}} = 0.50$ (also two significant figures)

(c) 36 mm (to the nearest millimeter)

(d) 6 mm

(e) 2.0 (two significant figures)

EVALUATE: The length of the rectangle is known only to the nearest mm, so the answers in parts **(c)** and (d) are known only to the nearest mm.

1.18. **IDENTIFY:** Estimate the number of people and then use the estimates given in the problem to calculate the number of gallons.

SET UP: Estimate 3×10^8 people, so 2×10^8 cars.

EXECUTE: (Number of cars × miles/car day)/(mi/gal) = gallons/day

$$(2\times10^8\text{ cars}\times10000\text{ mi/yr/car}\times1\text{ yr/365 days})/(20\text{ mi/gal}) = 3\times10^8\text{ gal/day}$$

EVALUATE: The number of gallons of gas used each day approximately equals the population of the U.S.

1.26. **IDENTIFY:** Since she returns to the starting point, the vector sum of the four displacements must be zero.

SET UP: Call the three given displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$, and call the fourth displacement $\vec{D}$. $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$.

EXECUTE: The vector addition diagram is sketched in Figure 1.26. Careful measurement gives that $\vec{D}$ is 144 m, 41° south of west.

EVALUATE: $\vec{D}$ is equal in magnitude and opposite in direction to the sum $\vec{A}+\vec{B}+\vec{C}$.

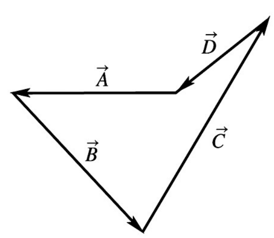
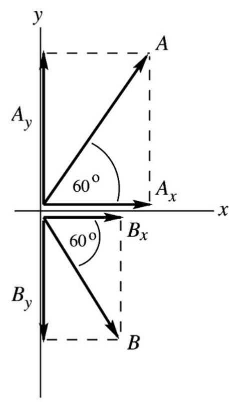


Figure 1.26

1.35. IDENTIFY: Vector addition problem. $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$.

SET UP: Find the x- and y-components of $\vec{A}$ and $\vec{B}$. Then the x- and y-components of the vector sum are calculated from the x- and y-components of $\vec{A}$ and $\vec{B}$.

EXECUTE:



$$A_x = A\cos(60.0°)$$
$$A_x = (2.80\ \text{cm})\cos(60.0°) = +1.40\ \text{cm}$$
$$A_y = A\sin(60.0°)$$
$$A_y = (2.80\ \text{cm})\sin(60.0°) = +2.425\ \text{cm}$$
$$B_x = B\cos(-60.0°)$$
$$B_x = (1.90\ \text{cm})\cos(-60.0°) = +0.95\ \text{cm}$$
$$B_y = B\sin(-60.0°)$$
$$B_y = (1.90\ \text{cm})\sin(-60.0°) = -1.645\ \text{cm}$$

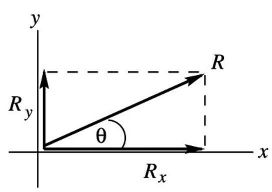
Note that the signs of the components correspond to the directions of the component vectors.

Figure 1.35a

(a) Now let $\vec{R} = \vec{A} + \vec{B}$.

$R_x = A_x + B_x = +1.40\ \text{cm} + 0.95\ \text{cm} = +2.35\ \text{cm}.$

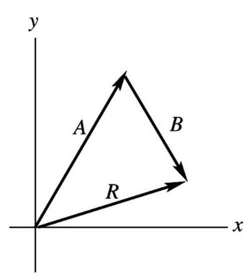
$R_y = A_y + B_y = +2.425\ \text{cm} - 1.645\ \text{cm} = +0.78\ \text{cm}.$



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(2.35\ \text{cm})^2 + (0.78\ \text{cm})^2}$$
$$R = 2.48\ \text{cm}$$
$$\tan\theta = \frac{R_y}{R_x} = \frac{+0.78\ \text{cm}}{+2.35\ \text{cm}} = +0.3319$$
$$\theta = 18.4°$$

Figure 1.35b

EVALUATE: The vector addition diagram for $\vec{R} = \vec{A} + \vec{B}$ is



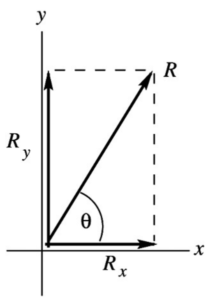
$\vec{R}$ is in the 1st quadrant, with $|R_y| < |R_x|$, in agreement with our calculation.

Figure 1.35c

(b) EXECUTE: Now let $\vec{R} = \vec{A} - \vec{B}$.

$R_x = A_x - B_x = +1.40 \text{ cm} - 0.95 \text{ cm} = +0.45 \text{ cm}.$

$R_y = A_y - B_y = +2.425 \text{ cm} + 1.645 \text{ cm} = +4.070 \text{ cm}.$



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(0.45 \text{ cm})^2 + (4.070 \text{ cm})^2}$$

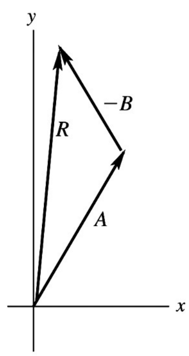
$$R = 4.09 \text{ cm}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{4.070 \text{ cm}}{0.45 \text{ cm}} = +9.044$$

$$\theta = 83.7°$$

Figure 1.35d

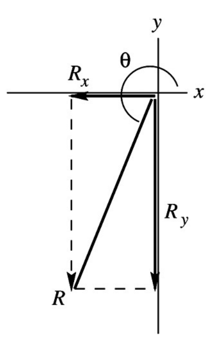
EVALUATE: The vector addition diagram for $\vec{R} = \vec{A} + (-\vec{B})$ is



$\vec{R}$ is in the 1st quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

Figure 1.35e

(c) EXECUTE:



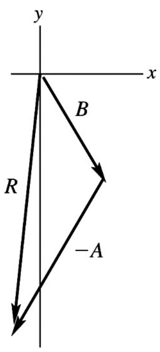
$\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$

$\vec{B} - \vec{A}$ and $\vec{A} - \vec{B}$ are equal in magnitude and opposite in direction.

$R = 4.09 \text{ cm}$ and $\theta = 83.7° + 180° = 264°$

Figure 1.35f

EVALUATE: The vector addition diagram for $\vec{R} = \vec{B} + (-\vec{A})$ is



$\vec{R}$ is in the 3rd quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

Figure 1.35g

1.38. **IDENTIFY:** Find A and B. Find the vector difference using components.

SET UP: Identify the x- and y-components and use $A = \sqrt{A_x^2 + A_y^2}$.

EXECUTE: **(a)** $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$; $A_x = +4.00$; $A_y = +7.00$.

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(4.00)^2 + (7.00)^2} = 8.06$. $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$; $B_x = +5.00$; $B_y = -2.00$;

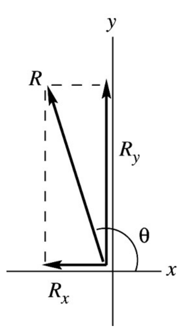
$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.00)^2 + (-2.00)^2} = 5.39$.

EVALUATE: Note that the magnitudes of $\vec{A}$ and $\vec{B}$ are each larger than either of their components.

EXECUTE: **(b)** $\vec{A} - \vec{B} = 4.00\hat{i} + 7.00\hat{j} - (5.00\hat{i} - 2.00\hat{j}) = (4.00 - 5.00)\hat{i} + (7.00 + 2.00)\hat{j}$.

$\vec{A} - \vec{B} = -1.00\hat{i} + 9.00\hat{j}$

(c) Let $\vec{R} = \vec{A} - \vec{B} = -1.00\hat{i} + 9.00\hat{j}$. Then $R_x = -1.00$, $R_y = 9.00$.



$R = \sqrt{R_x^2 + R_y^2}$

$R = \sqrt{(-1.00)^2 + (9.00)^2} = 9.06$.

$\tan\theta = \dfrac{R_y}{R_x} = \dfrac{9.00}{-1.00} = -9.00$

$\theta = -83.6° + 180° = 96.3°$.

Figure 1.38

EVALUATE: $R_x < 0$ and $R_y > 0$, so $\vec{R}$ is in the 2nd quadrant.

1.44.IDENTIFY: Target variable is the vector $\vec{A} \times \vec{B}$ expressed in terms of unit vectors.

SET UP: We are given $\vec{A}$ and $\vec{B}$ in unit vector form and can take the vector product using $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = 0$, $\hat{i} \times \hat{j} = \hat{k}$, and $\hat{j} \times \hat{i} = -\hat{k}$.

EXECUTE: $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$, $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$.

$\vec{A}\times\vec{B} = (4.00\hat{i} + 7.00\hat{j})\times(5.00\hat{i} - 2.00\hat{j}) = 20.0\hat{i}\times\hat{i} - 8.00\hat{i}\times\hat{j} + 35.0\hat{j}\times\hat{i} - 14.0\hat{j}\times\hat{j}$. But $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = 0$ and $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{i} = -\hat{k}$, so $\vec{A}\times\vec{B} = -8.00\hat{k} + 35.0(-\hat{k}) = -43.0\hat{k}$. The magnitude of $\vec{A}\times\vec{B}$ is 43.0.

EVALUATE: Sketch the vectors $\vec{A}$ and $\vec{B}$ in a coordinate system where the *xy*-plane is in the plane of the paper and the *z*-axis is directed out toward you. By the right-hand rule $\vec{A}\times\vec{B}$ is directed into the plane of the paper, in the $-z$-direction. This agrees with the above calculation that used unit vectors.

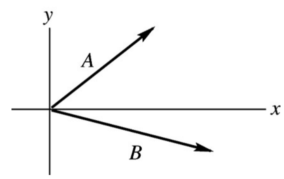


Figure 1.44

1.45. IDENTIFY: For all of these pairs of vectors, the angle is found from combining $\vec{A}\cdot\vec{B} = AB\cos\phi$ and

$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$, to give the angle ϕ as $\phi = \arccos\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right) = \arccos\left(\frac{A_xB_x + A_yB_y}{AB}\right)$.

SET UP: $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$ shows how to obtain the components for a vector written in terms of unit vectors.

EXECUTE: **(a)** $\vec{A}\cdot\vec{B} = -22$, $A = \sqrt{40}$, $B = \sqrt{13}$, and so $\phi = \arccos\left(\frac{-22}{\sqrt{40}\sqrt{13}}\right) = 165°$.

(b) $\vec{A}\cdot\vec{B} = 60$, $A = \sqrt{34}$, $B = \sqrt{136}$, $\phi = \arccos\left(\frac{60}{\sqrt{34}\sqrt{136}}\right) = 28°$.

(c) $\vec{A}\cdot\vec{B} = 0$ and $\phi = 90°$.

EVALUATE: If $\vec{A}\cdot\vec{B} > 0$, $0 \le \phi < 90°$. If $\vec{A}\cdot\vec{B} < 0$, $90° < \phi \le 180°$. If $\vec{A}\cdot\vec{B} = 0$, $\phi = 90°$ and the two vectors are perpendicular.

1.64. **IDENTIFY:** Solve for one of the vectors in the vector sum. Use components.

SET UP: Use coordinates for which $+x$ is east and $+y$ is north. The vector displacements are:

$\vec{A} = 2.00$ km, 0°of east; $\vec{B} = 3.50$ m, 45° south of east; and $\vec{R} = 5.80$ m, 0° east

EXECUTE: $C_x = R_x - A_x - B_x = 5.80\text{ km} - (2.00\text{ km}) - (3.50\text{ km})(\cos 45°) = 1.33\text{ km}$; $C_y = R_y - A_y - B_y$ $= 0\text{ km} - 0\text{ km} - (-3.50\text{ km})(\sin 45°) = 2.47\text{ km}$; $C = \sqrt{(1.33\text{ km})^2 + (2.47\text{ km})^2} = 2.81\text{ km}$;

$\theta = \tan^{-1}[(2.47\text{ km})/(1.33\text{ km})] = 61.7°$ north of east. The vector addition diagram in Figure 1.64 shows good qualitative agreement with these values.

EVALUATE: The third leg lies in the first quadrant since its *x* and *y* components are both positive.

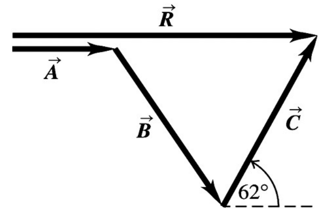


Figure 1.64

1.85.IDENTIFY and SET UP: The target variables are the components of $\vec{C}$. We are given $\vec{A}$ and $\vec{B}$. We also know $\vec{A}\cdot\vec{C}$ and $\vec{B}\cdot\vec{C}$, and this gives us two equations in the two unknowns C_x and C_y.

EXECUTE: $\vec{A}$ and $\vec{C}$ are perpendicular, so $\vec{A}\cdot\vec{C}=0$. $A_xC_x + A_yC_y = 0$, which gives $5.0C_x - 6.5C_y = 0$.

$\vec{B}\cdot\vec{C} = 15.0$, so $3.5C_x - 7.0C_y = 15.0$

We have two equations in two unknowns C_x and C_y. Solving gives $C_x = -8.0$ and $C_y = -6.1$.

EVALUATE: We can check that our result does give us a vector $\vec{C}$ that satisfies the two equations $\vec{A}\cdot\vec{C}=0$ and $\vec{B}\cdot\vec{C}=15.0$.