For problems 1, 2, and 3, do your work in the space provided, and write your final answer in the blank. Points will be deducted for the wrong units or wrong number of significant digits. Other than that, no partial credit will be awarded for incorrect answers.

1. (6 points) Energy is to be stored in a 75.0-kg flywheel in the shape of a uniform solid disk with radius $R = 1.25$ m. To prevent structural failure of the flywheel, the maximum allowed radial acceleration of a point on its rim is 3650 m/s^2. What is the maximum kinetic energy that can be stored in the flywheel?

Kinetic energy 8.55×10^4 J

$$E = \frac{1}{2} I \omega^2 = \frac{1}{2}\left(\frac{1}{2} M R^2\right)\omega^2 = \frac{1}{4} M v_{rim}^2$$

$$a_{MAX} = \frac{v_{rim-MAX}^2}{R} \Rightarrow E_{MAX} = \frac{1}{4} M R a_{MAX} = \frac{1}{4}(75.0\,kg)(1.25\,m)\left(3650\,\frac{m}{s^2}\right)$$

2. (7 points) Two identical 1.25-kg masses are pressed against (but not attached to) opposite ends of a light spring of force constant 1.95 N/cm, compressing the spring by 20.0 cm from its normal length. The masses are sitting on a horizontal, frictionless table, and initially are at rest. The system is then released. Find the speed of each mass when they are no longer in contact with the spring.

Speed 1.77 m/s

$P_i = 0 = P_f \Rightarrow$ Masses end with same speed

$$K_f = 2\left(\frac{1}{2} m v_f^2\right) = U_i = \frac{1}{2} k A^2 \Rightarrow$$

$$v_f = \sqrt{\frac{kA^2}{2m}} = \sqrt{\frac{(195\,\frac{N}{m})(0.2\,m)^2}{2\,(1.25\,kg)}} = 1.77\,\frac{m}{s}$$

3. (7 points) In the figure below, a small block is sitting on a horizontal, frictionless table. It is attached to a massless cord passing through a hole in the surface. The block is originally moving in a circular path at a distance of 0.400 m from the hole with an angular speed of 1.80 rad/s. The cord is then pulled from below, shortening the radius of the circle to 0.150 m. What is the angular speed of the block after the cord has been shortened?

Angular speed $12.8\,\frac{rad}{s}$

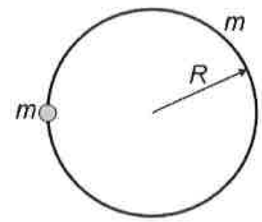
$L_i = L_f \Rightarrow I_i \omega_i = I_f \omega_f$

$$\Rightarrow \omega_f = \frac{\cancel{m} R_i^2}{\cancel{m} R_f^2}\omega_i = \left(\frac{0.400\,\cancel{m}}{0.150\,\cancel{m}}\right)^2 1.80\,\frac{rad}{s} = 12.8\,\frac{rad}{s}$$

For problems 4, 5, 6, and 7, do your work in the space provided, and write your final answer in the blank. For these problems, partial credit will be awarded where appropriate, based on the work that you show.

4. (20 points) A uniform, solid disk with mass m and radius R is pivoted about a horizontal, frictionless axis through its center. A small object of the same mass m is glued to the rim of the disk. If the disk is released from rest with the small object at the end of a horizontal radius, as shown in the figure below, find the angular speed when the small object is directly below the axis.

Angular speed $\sqrt{4g/3R}$



$$K_f + U_f = \cancel{K_i}^{\,0} + U_i$$

$$U_i = mgR$$

$$U_f = 0$$

$$K_f = \frac{1}{2} I \omega_f^2 = \frac{1}{2}\left(\frac{1}{2}mR^2 + mR^2\right)\omega_f^2 = \frac{1}{2}\,\frac{3}{2}\,mR^2\omega_f^2$$

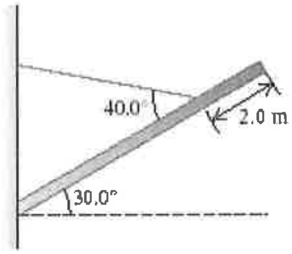
$$\Rightarrow \frac{3}{4}\cancel{m}R^{\cancel{2}}\omega_f^2 = \cancel{m}gR \Rightarrow \omega_f^2 = \frac{4g}{3R}$$

$$\Rightarrow \omega_f = \sqrt{\frac{4g}{3R}}$$

5. (20 points) A uniform 9.00-m, 550.-kg beam is hinged to a wall and supported by a thin cable attached 2.00 m from the free end of the beam, as shown in the figure below. The beam is supported at an angle of 30.0° above the horizontal.

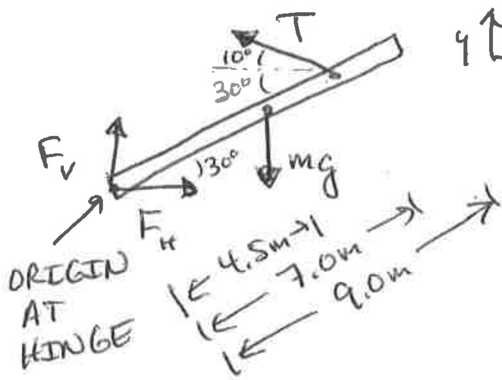
(a) What is the tension in the cable?

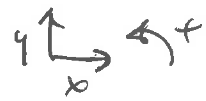
(b) What force does the beam exert on the hinge that holds it to the wall?



Tension 4670 N (zero not significant)

Force $(-4.60 \times 10^3 N)\hat{i} + (-4.58 \times 10^3 N)\hat{j}$





① $F_x = F_H - T\cos 10° = 0$

② $F_y = F_V + T\sin 10° - mg = 0$

③ $\tau = (7.0\,m)\,T\sin 140° - (4.5\,m)\,mg\sin 120° = 0$

③ $\Rightarrow T = \dfrac{(4.5\,m)(550\,kg)(9.80\,\frac{m}{s^2})\sin 120°}{(7.0\,m)\sin 140°} = 4670\,N$

① $\Rightarrow F_H = T\cos 10° = 4600\,N$ (last 0 not sig)

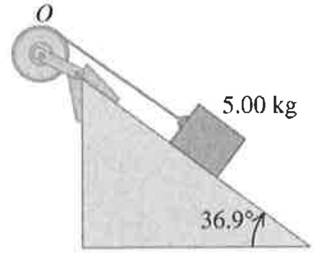
② $\Rightarrow F_V = (550\,kg)(9.80\,\frac{m}{s^2}) - (4670\,N)\sin 10°$

$= 4580\,N$ (zero not sig)

$\Rightarrow \vec{F}(\text{on beam}) = (4.60 \times 10^3 N)\hat{i} + (4.58 \times 10^3 N)\hat{j}$

$\vec{F}(\text{on hinge}) = -\vec{F}(\text{on beam})$

6. (20 points) A block with mass $m = 5.00$ kg slides down a surface inclined at 36.9^0 to the horizontal, as shown below. The coefficient of kinetic friction is 0.210. A string attached to the block is wrapped around a flywheel, mounted on a horizontal, frictionless, fixed axle at O. The flywheel is a uniform disk of mass 21.0 kg and radius 0.160 m. The string is wrapped around the outside of the flywheel so that it doesn't slip. Assume the string is attached to the block at just the right height so that it is parallel to the inclined surface.

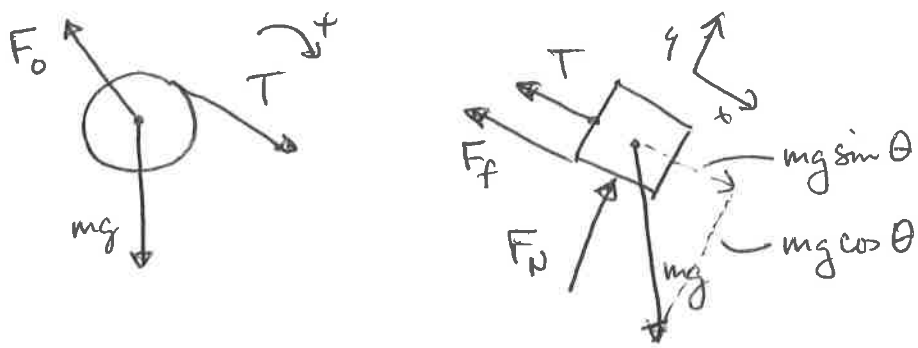


(a) What is the acceleration of the block down the plane?

(b) What is the tension in the string?

Acceleration $1.37\ \frac{m}{s^2}$

Tension 14.4 N



$$\tau = RT = \tfrac{1}{2} M_F R^2 \alpha$$

$$\Rightarrow \cancel{R} T = \tfrac{1}{2} M_F R^{\cancel{2}} \frac{a}{\cancel{R}}$$

$$\Rightarrow T = \tfrac{1}{2} M_F a$$

$$F_x = M_B g \sin\theta - T - F_f = M_B a$$

$$F_y = F_N - M_B g \cos\theta = 0$$

$$\Rightarrow F_N = M_B g \cos\theta$$

$$\Rightarrow F_f = \mu_K M_B g \cos\theta$$

$$\Rightarrow M_B g \sin\theta - \tfrac{1}{2} M_F a - \mu_K M_B g \cos\theta = M_B a$$

$$\Rightarrow \left(M_B + \tfrac{1}{2} M_F\right) a = M_B g \left(\sin\theta - \mu_K \cos\theta\right)$$

$$\Rightarrow a = \frac{M_B g \left(\sin\theta - \mu_K \cos\theta\right)}{M_B + \tfrac{1}{2} M_F} = 1.37\ \frac{m}{s^2}$$

$$\Rightarrow T = \tfrac{1}{2}(21.0\,\text{kg})\left(1.37\ \frac{m}{s^2}\right) = 14.4\ \text{N}$$

7. (20 points) To protect their young in the nest, peregrine falcons will fly into birds of prey (such as ravens) at high speed. In one such episode, a 550.-gm falcon flying at 21.0 m/s hits a 1.60-kg raven flying at 8.00 m/s. The falcon hits the raven at right angles to its original path and bounces back at 5.00 m/s. (These figures were made up by your professor while typing this exam.)

   (a) By what angle will the falcon change the raven's direction of motion?

   (b) What is the raven's speed right after the collision?

   Now assume that, instead of bouncing backwards, the falcon grips the raven with his talons, so the two birds fly off together.

   (c) By what angle will the falcon change the raven's direction of motion in this case?

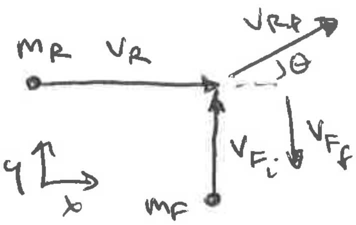
   (d) What will be the speed of the two birds right after the collision?

Angle (a) 48.2°

Speed (b) 12.0 m/s

Angle (c) 42.1°

Speed (d) 8.02 m/s



$$\vec{P}_i = \vec{P}_f \Rightarrow P_{x_i} = m_R v_R = P_{x_f} = m_R v_{R_f} \cos\theta$$

$$P_{y_i} = m_F v_{F_i} = P_{y_f} = m_R v_{R_f} \sin\theta - m_F v_{F_f}$$

$$\Rightarrow v_{R_f}\cos\theta = v_{R_{fx}} = 8.00\,\tfrac{m}{s}$$

$$v_{R_f}\sin\theta = v_{R_{fy}} = \frac{m_F(v_{F_i}+v_{F_f})}{m_R} = \frac{(0.55\,kg)(26\,\tfrac{m}{s})}{1.60\,kg} = 8.94\,\tfrac{m}{s}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{8.94\,\tfrac{m}{s}}{8.00\,\tfrac{m}{s}}\right) = 48.2^\circ$$

$$v_{R_f} = \sqrt{(8.00\,\tfrac{m}{s})^2 + (8.94\,\tfrac{m}{s})^2} = 12.0\,\tfrac{m}{s}$$

(c)+(d):

$$P_{x_i} = m_R v_R = P_{x_f} = (m_R + m_F) v_f \cos\theta \Rightarrow v_{f_x} = \frac{m_R}{m_R + m_F} v_R = 5.95\,\tfrac{m}{s}$$

$$P_{y_i} = m_F v_F = P_{y_f} = (m_R + m_F) v_f \sin\theta \Rightarrow v_{f_y} = \frac{m_F}{m_R + m_F} v_F = 5.37\,\tfrac{m}{s}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{5.37\,\tfrac{m}{s}}{5.95\,\tfrac{m}{s}}\right) = 42.1^\circ \qquad v_F = \sqrt{(5.95\,\tfrac{m}{s})^2 + (5.37\,\tfrac{m}{s})^2} = 8.02\,\tfrac{m}{s}$$

For problems 1, 2, and 3, do your work in the space provided, and write your final answer in the blank. Points will be deducted for the wrong units or wrong number of significant digits. Other than that, no partial credit will be awarded for incorrect answers.

1. (6 points) Energy is to be stored in a 65.0-kg flywheel in the shape of a uniform solid disk with radius $R = 1.15$ m. To prevent structural failure of the flywheel, the maximum allowed radial acceleration of a point on its rim is 3650 m/s^2. What is the maximum kinetic energy that can be stored in the flywheel?

Kinetic energy $6.82 * 10^4$ J

$$E_{MAX} = \frac{1}{2} I \omega_{MAX}^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_{MAX}^2$$

$$R\omega_{MAX} = v_{RIM\text{-}MAX} \Rightarrow E_{MAX} = \frac{1}{4} M v_{RIM\text{-}MAX}^2 = \frac{1}{4} M R a_{MAX}$$

$$a_{MAX} = \frac{(v_{RIM\text{-}MAX})^2}{R} \Rightarrow v_{RIM\text{-}MAX}^2 = a_{MAX} R$$

2. (7 points) Two identical 1.75-kg masses are pressed against (but not attached to) opposite ends of a light spring of force constant 1.65 N/cm, compressing the spring by 20.0 cm from its normal length. The masses are sitting on a horizontal, frictionless table, and initially are at rest. The system is then released. Find the speed of each mass when they are no longer in contact with the spring.

Speed $1.37 \frac{m}{s}$

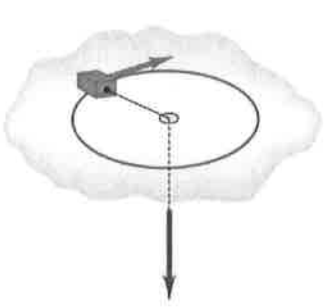
$P_i = 0 = P_f \Rightarrow$ Masses end with same v

$$K_f = 2\left(\frac{1}{2} M v_f^2\right) = U_i = \frac{1}{2} k A^2$$

$$\Rightarrow v_f = \sqrt{\frac{kA^2}{2m}} = \sqrt{\frac{\left(165 \frac{N}{m}\right)(0.2m)^2}{2 * 1.75 kg}} = 1.37 \frac{m}{s}$$

3. (7 points) In the figure below, a small block is sitting on a horizontal, frictionless table. It is attached to a massless cord passing through a hole in the surface. The block is originally moving in a circular path at a distance of 0.500 m from the hole with an angular speed of 1.90 rad/s. The cord is then pulled from below, shortening the radius of the circle to 0.150 m. What is the angular speed of the block after the cord has been shortened?

Angular speed $21.1 \frac{rad}{s}$



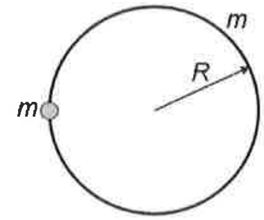
$$L_i = L_f \Rightarrow I_i \omega_i = I_f \omega_f \Rightarrow \cancel{m} R_i^2 \omega_i = \cancel{m} R_f^2 \omega_f$$

$$\Rightarrow \omega_f = \left(\frac{0.500 \cancel{m}}{0.150 \cancel{m}}\right)^2 1.90 \frac{rad}{s} = 21.1 \frac{rad}{s}$$

B

For problems 4, 5, 6, and 7, do your work in the space provided, and write your final answer in the blank. For these problems, partial credit will be awarded where appropriate, based on the work that you show.

4. (20 points) A uniform, solid disk with mass m and radius R is pivoted about a horizontal, frictionless axis through its center. A small object of the same mass m is glued to the rim of the disk. If the disk is released from rest with the small object at the end of a horizontal radius, as shown in the figure below, find the angular speed when the small object is directly below the axis.



Angular speed $\sqrt{4g/3R}$

$$K_f + U_f = \cancel{K_i}^{\,0} + U_i$$

$$U_i = mgR \qquad U_f = 0$$

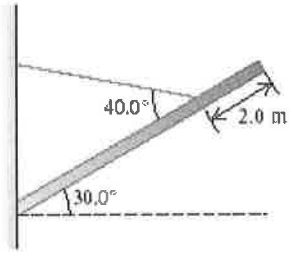
$$K_f = \frac{1}{2} I \omega_f^2 = \frac{1}{2}\left(\frac{1}{2}mR^2 + mR^2\right)\omega_f^2 = \frac{1}{2}\,\frac{3}{2} mR^2 \omega_f^2$$

$$\Rightarrow \frac{3}{4}\cancel{m}R^{\cancel{2}}\omega_f^2 = \cancel{m}g\cancel{R} \Rightarrow \omega_f^2 = \frac{4g}{3R} \Rightarrow \omega_f = \sqrt{\frac{4g}{3R}}$$

5. (20 points) A uniform 7.00-m, 550.-kg beam is hinged to a wall and supported by a thin cable attached 2.00 m from the free end of the beam, as shown in the figure below. The beam is supported at an angle of 30.0^0 above the horizontal.

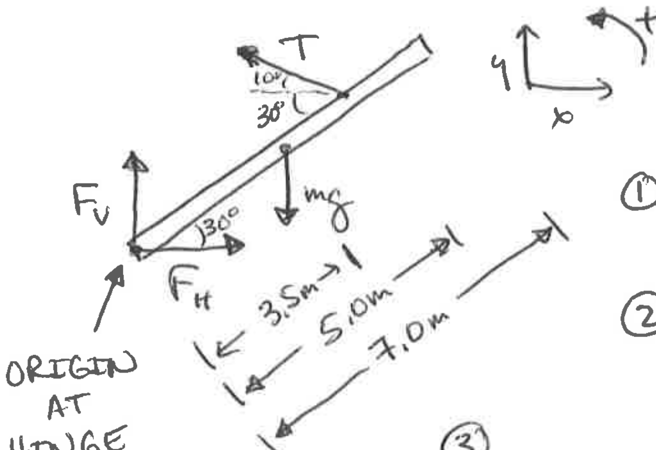
(a) What is the tension in the cable?

(b) What force does the beam exert on the hinge that holds it to the wall?



Tension $5.08 * 10^3\ N$

Force $(-5.01 * 10^3\ N)\hat{i} + (-4.51 * 10^3\ N)\hat{j}$



① $F_x = F_H - T\cos 10^\circ = 0$

② $F_y = F_V + T\sin 10^\circ - mg = 0$

③ $\tau = (5.0\,m)T\sin 140^\circ - (3.5\,m)mg\sin 120^\circ = 0$

③ $\Rightarrow T = \dfrac{(3.5\,m)(550\,kg)(9.80\,\frac{m}{s^2})\sin 120^\circ}{(5.0\,m)\sin 140^\circ} = 5080\ N$

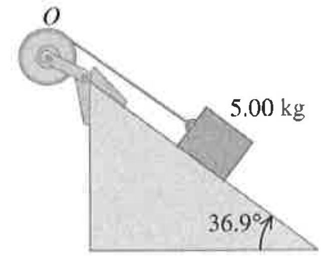
① $\Rightarrow F_H = T\cos 10^\circ = 5010\ N$

② $\Rightarrow F_V = (550\,kg)(9.80\,\frac{m}{s^2}) - T\sin 10^\circ = 4510\ N$

$\vec{F}(\text{on beam}) = (5010\,N)\hat{i} + (4510\,N)\hat{j}$

$\vec{F}(\text{on hinge}) = -\vec{F}(\text{on beam})$

6. (20 points) A block with mass $m = 5.00$ kg slides down a surface inclined at 36.9^0 to the horizontal, as shown below. The coefficient of kinetic friction is 0.190. A string attached to the block is wrapped around a flywheel, mounted on a horizontal, frictionless, fixed axle at O. The flywheel is a uniform disk of mass 19.0 kg and radius 0.180 m. The string is wrapped around the outside of the flywheel so that it doesn't slip. Assume the string is attached to the block at just the right height so that it is parallel to the inclined surface.

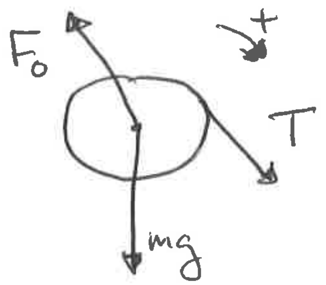


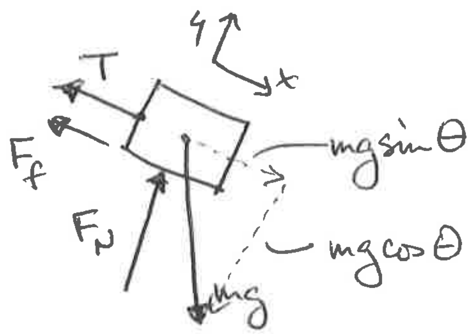
(a) What is the acceleration of the block down the plane?

(b) What is the tension in the string?

Acceleration $1.52\,\frac{m}{s^2}$

Tension 14.4 N





$$\tau = RT = \tfrac{1}{2} M_F R^2 \alpha$$

$$\Rightarrow \cancel{R}T = \tfrac{1}{2} M_F R^{\cancel{2}} \frac{a}{\cancel{R}}$$

$$\Rightarrow T = \tfrac{1}{2} M_F a$$

$$F_x = M_B g \sin\theta - T - F_f = M_B a$$

$$F_y = F_N - M_B g \cos\theta = 0 \Rightarrow F_N = M_B g \cos\theta$$

$$\Rightarrow F_f = \mu_K M_B g \cos\theta$$

$$\Rightarrow M_B g \sin\theta - \tfrac{1}{2} M_F a - \mu_K M_B g \cos\theta = M_B a$$

$$\Rightarrow \left(M_B + \tfrac{1}{2} M_F\right) a = M_B g \left(\sin\theta - \mu_K \cos\theta\right)$$

$$\Rightarrow a = \frac{M_B g \left(\sin\theta - \mu_K \cos\theta\right)}{M_B + \tfrac{1}{2} M_F} = 1.52\,\frac{m}{s^2}$$

$$\Rightarrow T = \tfrac{1}{2}(19.0\,\text{kg})\left(1.52\,\frac{m}{s^2}\right) = 14.4\,\text{N}$$

7. (20 points) To protect their young in the nest, peregrine falcons will fly into birds of prey (such as ravens) at high speed. In one such episode, a 550.-gm falcon flying at 22.0 m/s hits a 1.60-kg raven flying at 7.00 m/s. The falcon hits the raven at right angles to its original path and bounces back at 5.00 m/s. (These figures were made up by your professor while typing this exam.)

   (a) By what angle will the falcon change the raven's direction of motion?

   (b) What is the raven's speed right after the collision?

   Now assume that, instead of bouncing backwards, the falcon grips the raven with his talons, so the two birds fly off together.

   (c) By what angle will the falcon change the raven's direction of motion in this case?

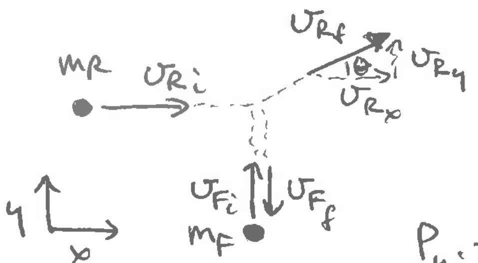
   (d) What will be the speed of the two birds right after the collision?

Angle (a) 53.0°

Speed (b) $11.6\ \frac{m}{s}$

Angle (c) 47.2°

Speed (d) $7.67\ \frac{m}{s}$



$$\vec{P}_i = \vec{P}_f \Rightarrow P_{x_i} = m_R v_{R_i} = P_f = m_R v_{R_x}$$

$$\Rightarrow v_{R_x} = v_{R_i} = 7.00\ \frac{m}{s}$$

$$P_{y_i} = m_F v_{F_i} = P_{y_f} = m_R v_{R_y} - m_F v_{F_f}$$

$$\Rightarrow v_{R_y} = \frac{m_F}{m_R}(v_{F_i} + v_{F_f}) = \frac{0.550\ kg}{1.60\ kg}\left(27.0\ \frac{m}{s}\right) = 9.28\ \frac{m}{s}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{9.28\ \cancel{m/s}}{7.00\ \cancel{m/s}}\right) = 53.0^\circ \qquad v_{R_f} = \sqrt{v_{R_x}^2 + v_{R_y}^2} = 11.6\ \frac{m}{s}$$

(c) + (d):

$$P_{x_i} = m_R v_{R_i} = P_{x_f} = (m_R + m_F) v_{R_x} \Rightarrow v_{R_x} = \frac{m_R}{m_R + m_F} v_{R_i} = 5.21\ \frac{m}{s}$$

$$P_{y_i} = m_F v_{F_i} = P_{y_f} = (m_R + m_F) v_{R_y} \Rightarrow v_{R_y} = \frac{m_F}{m_R + m_F} v_{F_i} = 5.63\ \frac{m}{s}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{5.63\ \cancel{m/s}}{5.21\ \cancel{m/s}}\right) = 47.2^\circ \qquad v_{R_f} = \sqrt{v_{R_x}^2 + v_{R_y}^2} = 7.67\ \frac{m}{s}$$