9.4 Q's

Tuesday, April 28, 2015 2:07 PM

9.4 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

1. Graphical Analysis The figures show the graphs of the first 10 terms, and the graphs of the first 10 terms of the sequence of partial sums, of each series.

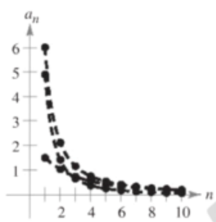
$$\sum_{n=1}^{\infty} \frac{6}{n^{3/2}}, \quad \sum_{n=1}^{\infty} \frac{6}{n^{3/2}+3}, \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{6}{n\sqrt{n^2+0.5}}$$

(a) Identify the series in each figure.

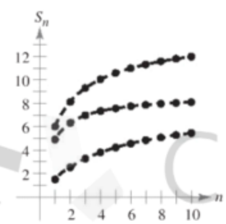
(b) Which series is a p-series? Does it converge or diverge?

(c) For the series that are not p-series, how do the magnitudes of the terms compare with the magnitudes of the terms of the p-series? What conclusion can you draw about the convergence or divergence of the series?

(d) Explain the relationship between the magnitudes of the terms of the series and the magnitudes of the terms of the partial sums.



Graphs of terms



Graphs of partial sums

2. Graphical Analysis The figures show the graphs of the first 10 terms, and the graphs of the first 10 terms of the sequence of partial sums, of each series.

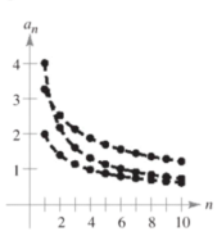
$$\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}}, \quad \sum_{n=1}^{\infty} \frac{2}{\sqrt{n}-0.5}, \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{4}{\sqrt{n+0.5}}$$

(a) Identify the series in each figure.

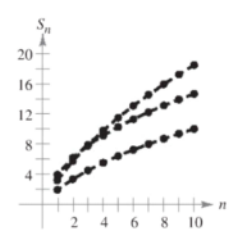
(b) Which series is a p-series? Does it converge or diverge?

(c) For the series that are not p-series, how do the magnitudes of the terms compare with the magnitudes of the terms of the p-series? What conclusion can you draw about the convergence or divergence of the series?

(d) Explain the relationship between the magnitudes of the terms of the series and the magnitudes of the terms of the partial sums.



Graphs of terms



Graphs of partial sums

Using the Direct Comparison Test In Exercises 3–12, use the Direct Comparison Test to determine the convergence or divergence of the series.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$

5. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

6. $\displaystyle\sum_{n=0}^{\infty} \frac{4^n}{5^n+3}$

7. $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n+1}$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$

9. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$

11. $\displaystyle\sum_{n=0}^{\infty} e^{-n^2}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{2^n-1}$

Using the Limit Comparison Test In Exercises 13–22, use the Limit Comparison Test to determine the convergence or divergence of the series.

13. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

14. $\displaystyle\sum_{n=1}^{\infty} \frac{5}{4^n+1}$

15. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$

16. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+1}{5^n+1}$

17. $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2-1}{3n^5+2n+1}$

18. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2(n+3)}$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n-1}}$

21. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k+1}, \quad k > 2$

22. $\displaystyle\sum_{n=1}^{\infty} \sin\frac{1}{n}$

Determining Convergence or Divergence In Exercises 23–30, test for convergence or divergence, using each test at least once. Identify which test was used.

(a) *n*th-Term Test
(b) Geometric Series Test
(c) *p*-Series Test
(d) Telescoping Series Test
(e) Integral Test
(f) Direct Comparison Test
(g) Limit Comparison Test

23. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n}$

24. $\displaystyle\sum_{n=0}^{\infty} 5\left(-\frac{4}{3}\right)^n$

25. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n+1}$

26. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^3-8}$

27. $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{3n-2}$

28. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$

30. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

31. Using the Limit Comparison Test Use the Limit Comparison Test with the harmonic series to show that the series $\Sigma\, a_n$ (where $0 < a_n < a_{n-1}$) diverges when $\displaystyle\lim_{n\to\infty} na_n$ is finite and nonzero.