9.3 Qs

Tuesday, April 28, 2015 2:09 PM

9.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Using the Integral Test In Exercises 1–22, confirm that the Integral Test can be applied to the series. Then use the Integral Test to determine the convergence or divergence of the series.

1. $\sum_{n=1}^{\infty} \frac{1}{n+3}$
2. $\sum_{n=1}^{\infty} \frac{2}{3n+5}$
3. $\sum_{n=1}^{\infty} \frac{1}{2^n}$
4. $\sum_{n=1}^{\infty} 3^{-n}$
5. $\sum_{n=1}^{\infty} e^{-n}$
6. $\sum_{n=1}^{\infty} ne^{-n/2}$
7. $\frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} + \frac{1}{26} + \cdots$
8. $\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \cdots$
9. $\frac{\ln 2}{2} + \frac{\ln 3}{3} + \frac{\ln 4}{4} + \frac{\ln 5}{5} + \frac{\ln 6}{6} + \cdots$
10. $\frac{\ln 2}{\sqrt{2}} + \frac{\ln 3}{\sqrt{3}} + \frac{\ln 4}{\sqrt{4}} + \frac{\ln 5}{\sqrt{5}} + \frac{\ln 6}{\sqrt{6}} + \cdots$
11. $\frac{1}{\sqrt{1}(\sqrt{1}+1)} + \frac{1}{\sqrt{2}(\sqrt{2}+1)} + \frac{1}{\sqrt{3}(\sqrt{3}+1)} + \cdots + \frac{1}{\sqrt{n}(\sqrt{n}+1)} + \cdots$
12. $\frac{1}{4} + \frac{2}{7} + \frac{3}{12} + \cdots + \frac{n}{n^2+3} + \cdots$
13. $\sum_{n=1}^{\infty} \frac{\arctan n}{n^2+1}$
14. $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$
15. $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$
16. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$
17. $\sum_{n=1}^{\infty} \frac{1}{(2n+3)^3}$
18. $\sum_{n=1}^{\infty} \frac{n+2}{n+1}$
19. $\sum_{n=1}^{\infty} \frac{4n}{2n^2+1}$
20. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+2}}$
21. $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$
22. $\sum_{n=1}^{\infty} \frac{n}{n^4+2n^2+1}$

Using the Integral Test In Exercises 23 and 24, use the Integral Test to determine the convergence or divergence of the series, where k is a positive integer.

23. $\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k+c}$
24. $\sum_{n=1}^{\infty} n^k e^{-n}$

Requirements of the Integral Test In Exercises 25–28, explain why the Integral Test does not apply to the series.

25. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$
26. $\sum_{n=1}^{\infty} e^{-n}\cos n$
27. $\sum_{n=1}^{\infty} \frac{2+\sin n}{n}$
28. $\sum_{n=1}^{\infty} \left(\frac{\sin n}{n}\right)^2$

Using the Integral Test In Exercises 29–32, use the Integral Test to determine the convergence or divergence of the p-series.

29. $\sum_{n=1}^{\infty} \frac{1}{n^3}$
30. $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$
31. $\sum_{n=1}^{\infty} \frac{1}{n^{1/4}}$
32. $\sum_{n=1}^{\infty} \frac{1}{n^5}$

Using a p-Series In Exercises 33–38, use Theorem 9.11 to determine the convergence or divergence of the p-series.

33. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$
34. $\sum_{n=1}^{\infty} \frac{3}{n^{5/3}}$
35. $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \cdots$
36. $1 + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \frac{1}{\sqrt[3]{16}} + \frac{1}{\sqrt[3]{25}} + \cdots$
37. $\sum_{n=1}^{\infty} \frac{1}{n^{1.04}}$
38. $\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$

39. **Numerical and Graphical Analysis** Use a graphing utility to find the indicated partial sum S_n and complete the table. Then use a graphing utility to graph the first 10 terms of the sequence of partial sums. For each series, compare the rate at which the sequence of partial sums approaches the sum of the series.

n	5	10	20	50	100
S_n					

(a) $\sum_{n=1}^{\infty} 3\left(\frac{1}{5}\right)^{n-1} = \frac{15}{4}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

40. **Numerical Reasoning** Because the harmonic series diverges, it follows that for any positive real number M, there exists a positive integer N such that the partial sum

$$\sum_{n=1}^{N} \frac{1}{n} > M.$$

(a) Use a graphing utility to complete the table.

M	2	4	6	8
N				

(b) As the real number M increases in equal increments, does the number N increase in equal increments? Explain.