

9.2

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9.2 Series and Convergence

- Understand the definition of a convergent infinite series.
- Use properties of infinite geometric series.
- Use the *n*th-Term Test for Divergence of an infinite series.

Infinite Series

One important application of infinite sequences is in representing “infinite summations.” Informally, if $\{a_n\}$ is an infinite sequence, then

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots \quad \text{Infinite Series}$$

REMARK As you study this chapter, it is important to distinguish between an infinite series and a sequence. A sequence is an ordered collection of numbers

$$a_1, a_2, a_3, \dots, a_n, \dots$$

whereas a series is an infinite sum of terms from a sequence

$$a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

is an **infinite series** (or simply a **series**). The numbers a_1, a_2, a_3 , and so on are the **terms** of the series. For some series, it is convenient to begin the index at $n = 0$ (or some other integer). As a typesetting convention, it is common to represent an infinite series as $\sum a_n$. In such cases, the starting value for the index must be taken from the context of the statement.

To find the sum of an infinite series, consider the **sequence of partial sums** listed below.

$$\begin{aligned} S_1 &= a_1 \\ S_2 &= a_1 + a_2 \\ S_3 &= a_1 + a_2 + a_3 \\ S_4 &= a_1 + a_2 + a_3 + a_4 \\ S_5 &= a_1 + a_2 + a_3 + a_4 + a_5 \\ &\vdots \\ S_n &= a_1 + a_2 + a_3 + \cdots + a_n \end{aligned}$$

If this sequence of partial sums converges, then the series is said to converge and has the sum indicated in the next definition.

INFINITE SERIES

The study of infinite series was considered a novelty in the fourteenth century. Logician Richard Suiseth, whose nickname was Calculator, solved this problem.

If throughout the first half of a given time interval a variation continues at a certain intensity, throughout the next quarter of the interval at double the intensity, throughout the following eighth at triple the intensity and so ad infinitum; then the average intensity for the whole interval will be the intensity of the variation during the second subinterval (or double the intensity). This is the same as saying that the sum of the infinite series

$$\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \cdots + \frac{n}{2^n} + \cdots$$

is 2.

Definitions of Convergent and Divergent Series

For the infinite series $\sum_{n=1}^{\infty} a_n$, the *n*th partial sum is

$$S_n = a_1 + a_2 + \cdots + a_n.$$

If the sequence of partial sums $\{S_n\}$ converges to S , then the series $\sum_{n=1}^{\infty} a_n$ **converges**. The limit S is called the **sum of the series**.

$$S = a_1 + a_2 + \cdots + a_n + \cdots \quad S = \sum_{n=1}^{\infty} a_n$$

If $\{S_n\}$ diverges, then the series **diverges**.

As you study this chapter, you will see that there are two basic questions involving infinite series.

- Does a series converge or does it diverge?
- When a series converges, what is its sum?

These questions are not always easy to answer, especially the second one.