9.10 Q's

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9.10 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Finding a Taylor Series In Exercises 1–12, use the definition of Taylor series to find the Taylor series, centered at c, for the function.

1. $f(x) = e^{2x}, \quad c = 0$
2. $f(x) = e^{-4x}, \quad c = 0$
3. $f(x) = \cos x, \quad c = \frac{\pi}{4}$
4. $f(x) = \sin x, \quad c = \frac{\pi}{4}$
5. $f(x) = \frac{1}{x}, \quad c = 1$
6. $f(x) = \frac{1}{1-x}, \quad c = 2$
7. $f(x) = \ln x, \quad c = 1$
8. $f(x) = e^{x}, \quad c = 1$
9. $f(x) = \sin 3x, \quad c = 0$
10. $f(x) = \ln(x^2 + 1), \quad c = 0$
11. $f(x) = \sec x, \quad c = 0$ (first three nonzero terms)
12. $f(x) = \tan x, \quad c = 0$ (first three nonzero terms)

Proof In Exercises 13–16, prove that the Maclaurin series for the function converges to the function for all x.

13. $f(x) = \cos x$
14. $f(x) = e^{-2x}$
15. $f(x) = \sinh x$
16. $f(x) = \cosh x$

Using a Binomial Series In Exercises 17–26, use the binomial series to find the Maclaurin series for the function.

17. $f(x) = \frac{1}{(1+x)^2}$
18. $f(x) = \frac{1}{(1+x)^4}$
19. $f(x) = \frac{1}{\sqrt{1-x}}$
20. $f(x) = \frac{1}{\sqrt{1-x^2}}$
21. $f(x) = \frac{1}{\sqrt{4+x^2}}$
22. $f(x) = \frac{1}{(2+x)^3}$
23. $f(x) = \sqrt{1+x}$
24. $f(x) = \sqrt[4]{1+x}$
25. $f(x) = \sqrt{1+x^2}$
26. $f(x) = \sqrt{1+x^3}$

Finding a Maclaurin Series In Exercises 27–40, find the Maclaurin series for the function. Use the table of power series for elementary functions on page 670.

27. $f(x) = e^{x^2/2}$
28. $g(x) = e^{-3x}$
29. $f(x) = \ln(1+x)$
30. $f(x) = \ln(1+x^2)$
31. $g(x) = \sin 3x$
32. $f(x) = \sin \pi x$
33. $f(x) = \cos 4x$
34. $f(x) = \cos \pi x$
35. $f(x) = \cos x^{3/2}$
36. $g(x) = 2 \sin x^3$
37. $f(x) = \frac{1}{2}(e^x - e^{-x}) = \sinh x$
38. $f(x) = e^x + e^{-x} = 2 \cosh x$
39. $f(x) = \cos^2 x$
40. $f(x) = \sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$

    $\left(\textit{Hint:}\text{ Integrate the series for } \frac{1}{\sqrt{x^2+1}}.\right)$

Finding a Maclaurin Series In Exercises 41–44, find the Maclaurin series for the function. (See Examples 7 and 8.)

41. $f(x) = x \sin x$
42. $h(x) = x \cos x$
43. $g(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$
44. $f(x) = \begin{cases} \frac{\arcsin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

Verifying a Formula In Exercises 45 and 46, use a power series and the fact that $i^2 = -1$ to verify the formula.

45. $g(x) = \frac{1}{2i}(e^{ix} - e^{-ix}) = \sin x$
46. $g(x) = \frac{1}{2}(e^{ix} + e^{-ix}) = \cos x$

Finding Terms of a Maclaurin Series In Exercises 47–52, find the first four nonzero terms of the Maclaurin series for the function by multiplying or dividing the appropriate power series. Use the table of power series for elementary functions on page 670. Use a graphing utility to graph the function and its corresponding polynomial approximation.

47. $f(x) = e^x \sin x$
48. $g(x) = e^x \cos x$
49. $h(x) = \cos x \ln(1+x)$
50. $f(x) = e^x \ln(1+x)$
51. $g(x) = \frac{\sin x}{1+x}$
52. $f(x) = \frac{e^x}{1+x}$

Finding a Maclaurin Series In Exercises 53 and 54, find a Maclaurin series for $f(x)$.

53. $f(x) = \int_0^x (e^{-t^2} - 1)\, dt$
54. $f(x) = \int_0^x \sqrt{1+t^3}\, dt$

Verifying a Sum In Exercises 55–58, verify the sum. Then use a graphing utility to approximate the sum with an error of less than 0.0001.

55. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = \ln 2$
56. $\sum_{n=0}^{\infty} (-1)^n \left[\frac{1}{(2n+1)!}\right] = \sin 1$
57. $\sum_{n=0}^{\infty} \frac{2^n}{n!} = e^2$
58. $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{1}{n!}\right) = \frac{e-1}{e}$

Finding a Limit In Exercises 59–62, use the series representation of the function f to find $\lim_{x \to 0} f(x)$ (if it exists).

59. $f(x) = \frac{1 - \cos x}{x}$
60. $f(x) = \frac{\sin x}{x}$
61. $f(x) = \frac{e^x - 1}{x}$
62. $f(x) = \frac{\ln(x+1)}{x}$