

9.1

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9.1 Sequences

- List the terms of a sequence.
- Determine whether a sequence converges or diverges.
- Write a formula for the n th term of a sequence.
- Use properties of monotonic sequences and bounded sequences.

Exploration

Finding Patterns Describe a pattern for each of the sequences listed below. Then use your description to write a formula for the n th term of each sequence. As n increases, do the terms appear to be approaching a limit? Explain your reasoning.

- a. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$
- b. $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \dots$
- c. $10, \frac{10}{3}, \frac{10}{6}, \frac{10}{10}, \frac{10}{15}, \dots$
- d. $\frac{1}{4}, \frac{4}{9}, \frac{9}{16}, \frac{16}{25}, \frac{25}{36}, \dots$
- e. $\frac{3}{7}, \frac{5}{10}, \frac{7}{13}, \frac{9}{16}, \frac{11}{19}, \dots$

Sequences

In mathematics, the word “sequence” is used in much the same way as it is in ordinary English. Saying that a collection of objects or events is *in sequence* usually means that the collection is ordered in such a way that it has an identified first member, second member, third member, and so on.

Mathematically, a **sequence** is defined as a function whose domain is the set of positive integers. Although a sequence is a function, it is common to represent sequences by subscript notation rather than by the standard function notation. For instance, in the sequence



1 is mapped onto a_1 , 2 is mapped onto a_2 , and so on. The numbers

$$a_1, a_2, a_3, \dots, a_n, \dots$$

are the **terms** of the sequence. The number a_n is the **n th term** of the sequence, and the entire sequence is denoted by $\{a_n\}$. Occasionally, it is convenient to begin a sequence with a_0 , so that the terms of the sequence become $a_0, a_1, a_2, a_3, \dots, a_n, \dots$ and the domain is the set of nonnegative integers.

EXAMPLE 1 Listing the Terms of a Sequence

a. The terms of the sequence $\{a_n\} = \{3 + (-1)^n\}$ are

$$3 + (-1)^1, 3 + (-1)^2, 3 + (-1)^3, 3 + (-1)^4, \dots$$

$$2, \quad 4, \quad 2, \quad 4, \quad \dots$$

b. The terms of the sequence $\{b_n\} = \left\{\frac{n}{1-2n}\right\}$ are

$$\frac{1}{1-2 \cdot 1}, \frac{2}{1-2 \cdot 2}, \frac{3}{1-2 \cdot 3}, \frac{4}{1-2 \cdot 4}, \dots$$

$$-1, \quad -\frac{2}{3}, \quad -\frac{3}{5}, \quad -\frac{4}{7}, \quad \dots$$

c. The terms of the sequence $\{c_n\} = \left\{\frac{n^2}{2^n - 1}\right\}$ are

$$\frac{1^2}{2^1 - 1}, \frac{2^2}{2^2 - 1}, \frac{3^2}{2^3 - 1}, \frac{4^2}{2^4 - 1}, \dots$$

$$\frac{1}{1}, \quad \frac{4}{3}, \quad \frac{9}{7}, \quad \frac{16}{15}, \quad \dots$$

d. The terms of the **recursively defined** sequence $\{d_n\}$, where $d_1 = 25$ and $d_{n+1} = d_n - 5$, are

$$25, \quad 25 - 5 = 20, \quad 20 - 5 = 15, \quad 15 - 5 = 10, \dots$$

•• **REMARK** Some sequences are defined recursively. To define a sequence recursively, you need to be given one or more of the first few terms. All other terms of the sequence are then defined using previous terms, as shown in Example 1(d).