MATH 132 **FINAL EXAM** **F12M132.4.1**

This exam should have 20 multiple choice questions, 5 points each. If you don't have a PENCIL to mark your card, please ask to borrow one from your proctor. Write your ID NUMBER (not your SS number) in the six boxes on the side of your answer card, and then shade in the corresponding numbers. The needed data for trigonometric functions and power series is contained on the last page.

1) Use substitution to evaluate $\int_0^{\pi/4} \tan^3(x) \cdot sec^2(x)\, dx$

$A)$ 0
$B)$ 0.15
$C)$ 0.25
$D)$ 0.35
$E)$ 0.45
$F)$ 0.55
$G)$ 0.65
$H)$ 0.75
I) 0.85
$J)$ 1

2) Use integration by parts to evaluate $\int_1^e x^2 ln(x)\, dx.$

$A)$ 4.08

$B)$ 4.16

$C)$ 4.28

$D)$ 4.34

$E)$ 4.42

$F)$ 4.57

$G)$ 4.69

$H)$ 4.74

$I)$ 4.82

$J)$ 4.93

3) Using partial fractions, find a solution to $\int \frac{-1}{x^2-x}\,dx.$ $(x > 1)$

A) $arctan(2x-1)$
B) $(x^2-x)^{-2}$
C) $ln(x^2-x)$
D) $ln(x)+ln(x-1)$
E) $ln(\frac{x}{x-1})$
F) $\frac{ln(x)}{ln(x-1)}$
G) $ln(\frac{x}{\mathrm{x}-1})+x$
$H)$ $ln(\frac{x}{x-1})-\frac{1}{x}$
$I)$ $\frac{ln(x-1)}{x^2}$
$J)$ $\frac{x^2}{ln(x-1)}$

4) Find what becomes of the integral $\int \frac{x^2}{\sqrt{9-x^2}}\,dx,$ when you make the substitution $x = 3\,sin(\theta),\ \ -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$

A) $3\int sin(\theta)\,d\theta$
B)$3\int cos(\theta)\,d\theta$
C) $\int \frac{9}{sin(\theta)}\,d\theta$
D) $\int \frac{9}{cos(\theta)}\,d\theta$
E) $3\int sec^2(\theta)\,d\theta$
F) $3\int csc^2(\theta)\,d\theta$
G) $9\int sin^2(\theta)\,d\theta$
H) $9\int cos^2(\theta)\,d\theta$
I) $\int \sqrt{9-sin^2(\theta)}\ d\theta$
J) $\int \sqrt{9-cos^2(\theta)}\ d\theta$

5) Find the **area** of the region enclosed by the curve $y = 2 - x^2$ and the line $y = 2 - 2\,x.$

$A)$ 1
$B)$ $\frac{1}{2}$
$C)$ $\frac{3}{2}$
$D)$ $\frac{4}{3}$
$E)$ $\frac{7}{5}$
$F)$ $\frac{5}{3}$
$G)$ $\frac{11}{5}$
$H)$ $\frac{9}{2}$
$I)$ $\frac{14}{3}$
$J)$ $\frac{13}{5}$

6) Find the **volume** of the solid obtained by **rotating** the region enclosed by the curve $x = \frac{2}{y}$, the lines $y = 1$, $y = 4$, and the y-axis, about the **y-axis**.

$A)$ $\frac{\pi}{4}$
$B)$ $\frac{3\pi}{4}$
$C)$ π
$D)$ $\frac{5\pi}{4}$
$E)$ $\frac{7\pi}{4}$
$F)$ $\frac{9\pi}{2}$
$G)$ 2π
$H)$ $\frac{9\pi}{4}$
$I)$ 3π
$J)$ $\frac{4\pi}{3}$

7) Find the **arc length** of the curve $x = \frac{2}{3}y^{3/2}$, $0 \leq y \leq 3$.

$A)$ $\frac{4}{3}$
$B)$ $\frac{8}{3}$
$C)$ $\frac{14}{3}$
$D)$ $\frac{3}{2}$
$E)$ $\frac{7}{2}$
$F)$ $\frac{9}{2}$
$G)$ $\frac{10}{3}$
$H)$ $\frac{11}{2}$
$I)$ $\frac{20}{3}$
J) $\frac{17}{2}$

8) Find the value of the improper integral $\int_0^\infty \frac{x}{(x^2+2)^2}\, dx,$ if it converges.
$A)$ 0
B) 2
C) 4
D) 6
$E)$ 8
$F)$ $\frac{1}{6}$
$G)$ $\frac{1}{4}$
$H)$ $\frac{1}{3}$
$I)$ $\frac{2}{3}$
$J)$ $diverges$

9) Find the solution to the initial value differential equation $\frac{dy}{dt} = t^2 y,\ y(0) = 2.$

$A)\, y \;=\; 2\,e^{t^2/2}$
$B)\;\; y = \;2\,e^{3t^2}$
$C)\, y \;=\; 2\,e^{t^3/3}$
$D)\, y \;=\; (t + \sqrt{2})^2$
$E)\, y \;=\; ln(t^2 + 1) + 2$
$F)\, y \;=\; 2\,ln(t + e)$
$G)\, y \;=\; (t + 1)^{-2} + 2$
$H)\, y = (t + 2)$
$I)\, y \;=\; t^2 + \frac{1}{2}$
$J)\, y \;=\; \sqrt{t + 4}$

10) Find the sum of the infinite series $\sum_{n=1}^{\infty} \frac{2^n - 1}{3^n}$.

$A)\ \frac{5}{3}$
$B)\ \frac{5}{4}$
$C)\ \frac{3}{2}$
$D)\ \frac{9}{5}$
$E)\ \frac{1}{5}$
$F)\ \frac{5}{2}$
$G)\ \frac{2}{3}$
$H)\ \frac{1}{2}$
$I)\ \frac{1}{4}$
$J)\ \frac{2}{5}$

11) Approximate the sum $\sum_{n=0}^{\infty} (-1)^n \frac{1}{n!}$ with an error of less than 2 x 10^{-3} .

$A)$ 0.367
$B)$ 0.371
$C)$ 0.384
$D)$ 0.396
$E)$ 0.408
$F)$ 0.414
$G)$ 0.429
$H)$ 0.437
$I)$ 0.4458
$J)$ 0.452

12) Find the complete interval of convergence (either absolute or conditional) of the power series $\sum_{n=1}^{\infty} \frac{2^n (x-4)^n}{n \cdot 3^n}$.

A) $\frac{5}{2} < x < \frac{11}{2}$
$B)$ $\frac{5}{2} \leq x < \frac{11}{2}$
$C)$ $\frac{5}{2} < x \leq \frac{11}{2}$
$D)$ $\frac{5}{2} \leq x \leq \frac{11}{2}$
$E)$ $\frac{3}{2} < x < \frac{13}{2}$
$F)$ $\frac{3}{2} \leq x < \frac{13}{2}$
$G)$ $\frac{3}{2} < x \leq \frac{13}{2}$
$H)$ $\frac{3}{2} \leq x \leq \frac{13}{2}$
$I)$ $-\infty < x < \infty$
$J)$ $\{4\}$

13) Find the Taylor polynomial of order 2 for $f(x) = \frac{1}{x}$ at $x = 1$.

$A)\ 1 + (x-1) + (x-1)^2$
$B)\ 1 - (x-1) + (x-1)^2$
$C)\ 1 + (x-1) - (x-1)^2$
$D)\ 1 + \frac{1}{2}(x-1) + (x-1)^2$
$E)\ 1 - (x-1) + \frac{1}{2}(x-1)^2$
$F)\ 1 - \frac{1}{2}(x-1) + (x-1)^2$
$G)\ 1 + \frac{1}{2}(x-1) + \frac{3}{2}(x-1)^2$
$H)\ 1 - \frac{1}{2}(x-1) + \frac{3}{2}(x-1)^2$
$I)\ 1 + \frac{1}{2}(x-1) - \frac{3}{2}(x-1)^2$
$J)\ 1 - \frac{1}{2}(x-1) - \frac{3}{2}(x-1)^2$

14) If the Maclaurin series for $x \cdot e^{2x}$ is $\sum_{n=1}^{\infty} c_n x^n$, then find c_4.

$A)\ \frac{1}{2}$
$B)\ \frac{3}{2}$
$C)\ \frac{2}{3}$
$D)\ \frac{4}{3}$
$E)\ \frac{1}{4}$
$F)\ \frac{3}{4}$
$G)\ \frac{5}{2}$
$H)\ \frac{5}{3}$
$I)\ \frac{5}{6}$
$J)\ \frac{6}{5}$

15) Using the Maclaurin series for $ln(1+x)$, approximate $\int_0^{0.5} ln(1+x)\,dx$, with an error $\leq$ 0.002.

$A)$ 0.109375
$B)$ 0.210432
$C)$ 0.342871
$D)$ 0.465732
$E)$ 0.543876
$F)$ 0.623149
$G)$ 0.743987
$H)$ 0.841762
$I)$ 0.954129
$J)$ 1.097658

16) Find the Maclaurin series for the function $x^2 - x \cdot tan^{-1}(x)$.

$A)$ $\sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+2}}{2n+2}$
$B)$ $\sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+4}}{2n+2}$
$C)$ $\sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+2}}{2n+1}$
$D)$ $\sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+4}}{2n+3}$
$E)$ $\sum_{n=0}^{\infty} \frac{(-1)^n\, x^{3n-1}}{3n}$
$F)$ $\sum_{n=0}^{\infty} \frac{(-1)^n\, x^{3n+2}}{3n+1}$
$G)$ $\sum_{n=0}^{\infty} \frac{(-1)^n\, x^{3n+3}}{3n+2}$
$H)$ $\sum_{n=0}^{\infty} \frac{(-1)^n\, x^{3n+4}}{3n+3}$
$I)$ $\sum_{n=0}^{\infty} \frac{(-1)^n\, (x^2)^{2n+1}}{4n+1}$
$J)$ $\sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n+2}}{4n+2}$

17) Find the Taylor series for $f(x) = 1 + x + x^2$ centered at $a = 3.$

$A)$ $13\ + 5(x-3) + (x-3)^2$
$B)$ $12\ + 5(x-3) + 2(x-3)^2$
$C)$ $13\ + 6(x-3) + (x-3)^2$
$D)$ $12\ + 6(x-3) + 2(x-3)^2$
$E)$ $13\ + 7(x-3) + (x-3)^2$
$F)$ $13\ + 7(x-3) + 2(x-3)^2$
$G)$ $13\ + 8(x-3) + (x-3)^2$
$H)$ $12\ + 8(x-3) + 2(x-3)^2$
$I)$ $13\ + 9(x-3) + (x-3)^2$
$J)$ $12\ + 9(x-3) + 2(x-3)^2$

18) If $sin(x) = \sum_{n=0}^{\infty} c_n\ (x - \frac{\pi}{4})^n$, is the Taylor series for the function $sin(x)$, centered at $a = \frac{\pi}{4}$, then find the coeficient, c_2, of $(x - \frac{\pi}{4})^2$.

A) $-\frac{\sqrt{2}}{4}$
$B)$ $\frac{1}{6}$
$C)$ $-\frac{\sqrt{2}}{8}$
$D)$ $\frac{1}{10}$
$E)$ $-\frac{\sqrt{2}}{12}$
$F)$ $\frac{1}{14}$
$G)$ $-\frac{\sqrt{2}}{16}$
$H)$ $\frac{1}{18}$
$I)$ $-\frac{\sqrt{2}}{20}$
$J)$ $\frac{1}{22}$

19) From the formula of the Binomial Series, we get that

$(1-2x)^{\frac{1}{2}} = \sum_{n=0}^{\infty} c_n x^n$, with $c_3 =$:

A) 0
B) $\frac{1}{4}$
$C)\ -\frac{1}{4}$
$D)\ \frac{1}{2}$
$E)\ -\frac{1}{2}$
$F)\ \frac{3}{8}$
$G)\ -\frac{3}{8}$
$H)\ \frac{4}{27}$
$I)\ -\frac{4}{27}$
$J)\ 1$

20) Using the Alternating Series Estimation Theorem, and the Maclauren Series for e^x, estimate $\int_0^{0.1} e^{-x^2} dx$, with an error less than or equal to 1 x 10^{-6} .

$A)$ 0.923367
B) 0.932765
C) 0.943876
D) 0.957627
E) 0.963333
F) 0.976667
$G)$ 0.987433
$H)$ 0.996667
$I)$ 1.095233
$J)$ 1.136667