MATH 142 - CALCULUS II (SECTIONS 11-12)
TEST 1 – FEBRUARY 3, 2005

1	(10 *pts*)
2	(10 *pts*)
3	(30 *pts*)
4	(10 *pts*)
5	(15 *pts*)
6	(15 *pts*)
7	(10 *pts*)

Name: ______________________

Directions: No Calculators are allowed on Tests or Exams. To receive proper credit, you must show your work and *box* your final answer.

1. Compute the derivative of $F(x) := \int_{\sqrt{x}}^{2x} \sqrt{1+t^2}\, dt$

Let $G(x) = \int_0^x \sqrt{1+t^2}\,dt$, then by the Fundamental Theorem of Calculus $G'(x) = \sqrt{1+x^2}$. $F(x) = G(2x) - G(\sqrt{x})$

so application of the chain rule gives

$$F'(x) = G'(2x)\cdot(2x)' - G'(\sqrt{x})\cdot(\sqrt{x})'$$
$$= \left(\sqrt{1+(2x)^2}\right)(2) - \left(\sqrt{1+(\sqrt{x})^2}\right)\left(\tfrac{1}{2}x^{-1/2}\right)$$
$$= \boxed{2\sqrt{1+4x^2} - \tfrac{1}{2}x^{-1/2}\sqrt{1+x}}$$

2. (a) Express $(x+\sin x)^{\frac{1}{x}}$ in terms of the ln and exp functions.

$$(x+\sin x)^{1/x} = e^{\ln\left((x+\sin x)^{1/x}\right)}$$
$$= \boxed{e^{\frac{1}{x}\ln(x+\sin x)}}$$

(b) Differentiate $(x+\sin x)^{\frac{1}{x}}$.

$$\left(e^{\frac{1}{x}\cdot\ln(x+\sin x)}\right)' = e^{\frac{1}{x}\ln(x+\sin x)}\cdot\left(\tfrac{1}{x}\ln(x+\sin x)\right)'$$
$$= \boxed{e^{\frac{1}{x}\ln(x+\sin x)}\left[-1\cdot x^{-2}\ln(x+\sin x) + x^{-1}\left(\frac{1+\cos x}{x+\sin x}\right)\right]}$$