PRINTABLE VERSION

Quiz 12

You scored 70 out of 100

Question 1

Your answer is CORRECT.

Convert

$$y'' - 2t\,y' + 5y = \cos(2t)$$

into a system of first-order equations.

a) $[x_1' = x_2,\ \ x_2' = -5x_1 + 2tx_2 - 2\sin(2t)]$

b) $[x_1' = x_2,\ \ x_2' = 5x_1 + 2tx_2 + \cos(2t)]$

c) $[x_1' = x_2,\ \ x_2' = -5x_1 + 2tx_2 + \cos(2t)]$

d) $[x_1' = x_2,\ \ x_2' = -2tx_1 + 5x_2 - \cos(2t)]$

e) $[x_1' = x_2,\ \ x_2' = -2tx_1 - 5x_2 - 2\sin(2t)]$

f) None of the above.

Question 2

Your answer is CORRECT.

Convert

$$y'' + 6y = \sin(5t)$$

into a system of first-order equations.

a) $[x_1' = x_2,\ \ x_2' = -6x_1 + 5\cos(5t)]$

b) $[x_1' = x_2,\ \ x_2' = -6x_1 + \sin(5t)]$

c) $[x_1' = x_2,\ \ x_2' = -6x_2 + 5\cos(5t)]$

d) $[x_1' = x_2,\ \ x_2' = 6x_1 + \sin(5t)]$

e) $[x_1' = x_2,\ \ x_2' = 6x_2 - \sin(5t)]$

f) None of the above.

Question 3

Your answer is CORRECT.

Convert

$$y''' - 4y'' + y' + y = \cos(2t)$$

into a system of first-order equations.

a) $[x_1' = x_2,\ x_2' = x_3,\ x_3' = -x_1 - x_2 + 4x_3 + \cos(2t)]$

b) $[x_1' = x_2,\ x_2' = x_3,\ x_3' = x_1 + x_2 - 4x_3 + \cos(2t)]$

c) $[x_1' = x_2,\ x_2' = x_3,\ x_3' = x_1 + 4x_2 - x_3 + \cos(2t)]$

d) $[x_1' = x_2,\ x_2' = x_3,\ x_3' = -x_1 + 4x_2 - x_3 - \cos(2t)]$

e) $[x_1' = x_2,\ x_2' = x_3,\ x_3' = x_1 + x_2 - 4x_3 - \cos(2t)]$

f) None of the above.

Question 4

Your answer is CORRECT.

A matrix function *A* and a vector function **b** are given. Write the system of equations corresponding to **x** ' = *A*(*t*) **x** + **b** (*t*).

$$A(t) = \begin{bmatrix} -4 & 1 \\ 3 & 2 \end{bmatrix},\quad \mathbf{b}(t) = \begin{bmatrix} e^{4t} \\ e^{-2t} \end{bmatrix}$$

a) $[x_1' = -4x_1 + x_2 - e^{4t},\ x_2' = 3x_1 + 2x_2 - e^{-2t}]$

b) $[x_1' = -4x_1 + 3x_2 + e^{4t},\ x_2' = x_1 + 2x_2 + e^{-2t}]$

c) $[x_1' = -4x_1 + 3x_2 - e^{4t},\ x_2' = x_1 + 2x_2 - e^{-2t}]$

d) $[x_1' = -4x_1 + x_2 + e^{4t},\ x_2' = 3x_1 + 2x_2 + e^{-2t}]$

e) $[x_1' = 3x_1 + 2x_2 + e^{4t},\ x_2' = -4x_1 + x_2 + e^{-2t}]$

f) None of the above.

Question 5

Your answer is CORRECT.

A matrix function A and a vector function **b** are given. Write the system of equations corresponding to **x** ' = $A(t)$ **x** + **b** (t).

$$A(t) = \begin{bmatrix} t^4 & -t \\ \sin(t) & -2 \end{bmatrix}, \quad \mathbf{b}(t) = \begin{bmatrix} t-2 \\ 2 \end{bmatrix}$$

a) $\left[x_1' = t^4 x_1 + \sin(t)\, x_2 - t + 2,\;\; x_2' = -t x_1 - 2x_2 - 2\right]$

b) $\left[x_1' = t^4 x_1 - t x_2 - t + 2,\;\; x_2' = \sin(t)\, x_1 - 2x_2 - 2\right]$

c) $\left[x_1' = t^4 x_1 - t x_2 + t - 2,\;\; x_2' = \sin(t)\, x_1 - 2x_2 + 2\right]$

d) $\left[x_1' = t^4 x_1 + \sin(t)\, x_2 + t - 2,\;\; x_2' = -t x_1 - 2x_2 + 2\right]$

e) $\left[x_1' = \sin(t)\, x_1 - 2x_2 + t - 2,\;\; x_2' = t^4 x_1 - t x_2 + 2\right]$

f) None of the above.

Question 6

Your answer is CORRECT.

A matrix function A and a vector function **b** are given. Write the system of equations corresponding to **x** ' = $A(t)$ **x** + **b** (t).

$$A(t) = \begin{bmatrix} -2 & 1 & 4 \\ 2 & 3 & -3 \\ -4 & 1 & -3 \end{bmatrix}, \quad \mathbf{b}(t) = \begin{bmatrix} e^t \\ 2e^{-t} \\ e^{-2t} \end{bmatrix}$$

a) $\left[x_1' = -2x_1 + 2x_2 - 4x_3 - e^t,\;\; x_2' = x_1 + 3x_2 + x_3 - 2e^{-t},\;\; x_3' = 4x_1 - 3x_2 - 3x_3 - e^{-2t}\right]$

b) $\left[x_1' = -2x_1 + x_2 + 4x_3 - e^t,\;\; x_2' = 2x_1 + 3x_2 - 3x_3 - 2e^{-t},\;\; x_3' = -4x_1 + x_2 - 3x_3 - e^{-2t}\right]$

c) $\left[x_1' = -2x_1 + x_2 + 4x_3 + e^t,\;\; x_2' = 2x_1 + 3x_2 - 3x_3 + 2e^{-t},\;\; x_3' = -4x_1 + x_2 - 3x_3 + e^{-2t}\right]$

d) ○ $\left[x'_1 = -2x_1 + 2x_2 - 4x_3 + e^t,\ \ x'_2 = x_1 + 3x_2 + x_3 + 2e^{-t},\ \ x'_3 = 4x_1 - 3x_2 - 3x_3 + e^{-2t}\right]$

e) ○ $\left[x'_1 = -2x_1 - x_2 + 4x_3 - e^t,\ \ x'_2 = 2x_1 - 3x_2 - 3x_3 + 2e^{-t},\ \ x'_3 = -4x_1 - x_2 + 3x_3 + e^{-2t}\right]$

f) ○ None of the above.

Question 7

Your answer is CORRECT.

A matrix function A and a vector function **b** are given. Write the system of equations corresponding to $\mathbf{x}' = A(t)\,\mathbf{x} + \mathbf{b}(t)$.

$$A(t) = \begin{bmatrix} t^3 & -3t & t+2 \\ 2 & t-4 & 2t \\ 3t & 2 & t \end{bmatrix},\quad \mathbf{b}(t) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

a) ○ $\left[x'_1 = t^3x_1 + 2x_2 + 3tx_3 + 1,\ \ x'_2 = -3tx_1 + (t-4)x_2 + 2x_3 + 1,\ \ x'_3 = (t+2)x_1 + 2tx_2 + tx_3\right]$

b) ◉ $\left[x'_1 = t^3x_1 - 3tx_2 + (t+2)x_3 + 1,\ \ x'_2 = 2x_1 + (t-4)x_2 + 2tx_3 + 1,\ \ x'_3 = 3tx_1 + 2x_2 + tx_3\right]$

c) ○ $\left[x'_1 = t^3x_1 + 3tx_2 + (t+2)x_3 - 1,\ \ x'_2 = 2x_1 - (t-4)x_2 + 2tx_3 + 1,\ \ x'_3 = 3tx_1 - 2x_2 - tx_3\right]$

d) ○ $\left[x'_1 = t^3x_1 + 2x_2 + 3tx_3 - 1,\ \ x'_2 = -3tx_1 - (t-4)x_2 + 2x_3 - 1,\ \ x'_3 = -(t+2)x_1 + 2tx_2 + tx_3\right]$

e) ○ $\left[x'_1 = t^3x_1 + 3tx_2 + (t+2)x_3 - 1,\ \ x'_2 = 2x_1 + (t-4)x_2 - 2tx_3 - 1,\ \ x'_3 = 3tx_1 + 2x_2 + tx_3\right]$

f) ○ None of the above.

Question 8

Your answer is CORRECT.

Write the system in vector matrix form:

$$\left[x'_1 = 3x_1 - 2x_2 + \sin(t),\ \ x'_2 = -4x_1 - 4\cos(t)\right]$$

a) ○ $\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \sin(t) \\ -4\cos(t) \end{bmatrix}$

b) $\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} 3 & -2 & \sin(t) \\ -4 & 0 & -4\cos(t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

c) $\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} 3 & -4 & \sin(t) \\ -2 & 0 & -4\cos(t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

d) (selected) $\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \sin(t) \\ -4\cos(t) \end{bmatrix}$

e) $\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -4\cos(t) \\ \sin(t) \end{bmatrix}$

f) None of the above.

Question 9

Your answer is CORRECT.

Write the system in vector matrix form:

$$\left[x'_1 = e^{-2t} x_1 + 4 e^{t} x_2,\quad x'_2 = -e^{4t} x_1 - 3 e^{-t} x_2\right]$$

a) $\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} e^{-2t} & -e^{4t} \\ 4e^{t} & -3e^{-t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

b) $\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} e^{-2t} \\ -3e^{-t} \end{bmatrix}$

c) (selected) $\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} e^{-2t} & 4e^{t} \\ -e^{4t} & -3e^{-t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

d) $\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} e^{-2t} & e^{4t} \\ 4e^{t} & 3e^{-t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

e) $\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 4e^{t} \\ -3e^{-t} \end{bmatrix}$

f) None of the above.

Question 10

Your answer is CORRECT.

Write the system in vector matrix form:

$$\left[x_1' = 2x_1 + x_2 + 4x_3 + 3e^{5t},\ \ x_2' = -x_1 + 3x_2 - 2x_3 + \sin(4t),\ \ x_3' = -4x_1 + 3x_3 - 3t\right]$$

a) $$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 2 & -1 & -4 \\ 1 & 3 & 0 \\ 4 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 3e^{5t} \\ \sin(4t) \\ -3t \end{bmatrix}$$

b) $$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 2 & -1 & -4 \\ 1 & 3 & 0 \\ 4 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -3e^{5t} \\ -\sin(4t) \\ 3t \end{bmatrix}$$

c) $$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ -1 & 3 & -2 \\ -4 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 3e^{5t} \\ \sin(4t) \\ -3t \end{bmatrix}$$

d) $$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 2 & -1 & 4 \\ 1 & 3 & -2 \\ 4 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 3e^{5t} \\ \sin(4t) \\ -3t \end{bmatrix}$$

e) $$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ -1 & 3 & -2 \\ -4 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -3e^{5t} \\ -\sin(4t) \\ 3t \end{bmatrix}$$

f) None of the above.

Question 11

Your answer is CORRECT.

Which of the following is a solution to **x** ' = $A(t)$ **x** + **b** (t) if

$$A(t) = \begin{bmatrix} -3 & -1 \\ 7 & 3 \end{bmatrix}, \quad \mathbf{b}(t) = \begin{bmatrix} 0 \\ -3\cos(t) \end{bmatrix}$$

a) $\mathbf{v}(t) = \begin{bmatrix} \sin(t) \\ -\cos(t) - 3\sin(t) \end{bmatrix}$

b) $\mathbf{v}(t) = \begin{bmatrix} \sin(t) - 3\cos(t) \\ -\cos(t) \end{bmatrix}$

c) $\mathbf{v}(t) = \begin{bmatrix} -\sin(t) + 3\cos(t) \\ -\cos(t) \end{bmatrix}$

d) $\mathbf{v}(t) = \begin{bmatrix} -\cos(t) \\ -\sin(t) + 3\cos(t) \end{bmatrix}$

e) $\mathbf{v}(t) = \begin{bmatrix} \sin(t) - 3\cos(t) \\ -4\cos(t) \end{bmatrix}$

f) None of the above.

Question 12

Your answer is CORRECT.

Find the Wronskian of the following vector functions:

$$\mathbf{u}(t) = \begin{bmatrix} 5t - 3 \\ -t \end{bmatrix}, \quad \mathbf{v}(t) = \begin{bmatrix} -t + 3 \\ 5t \end{bmatrix}$$

a) $10t^2$

b) $-26t^2 - 12t$

c) $-24t^2 + 13t$

d) $24t^2 - 12t$

e) $-10t^2 - t$

f) None of the above.

Question 13

Your answer is INCORRECT.

Find the Wronskian of the following vector functions:

$$\mathbf{v}_1(t) = \begin{bmatrix} e^{3t} \\ -2e^{3t} \\ 4e^{3t} \end{bmatrix}, \quad \mathbf{v}_2(t) = \begin{bmatrix} e^{-t} \\ -3e^{-t} \\ 7e^{-t} \end{bmatrix}, \quad \mathbf{v}_3(t) = \begin{bmatrix} te^{3t} \\ e^{3t} - 2te^{3t} \\ 4e^{3t} + 4te^{3t} \end{bmatrix}$$

a) $-7e^{5t}$

b) $-7e^{6t}$

c) $-7e^{2t}$

d) $7e^{5t} + e^{t}$

e) $7e^{2t} - e^{t}$

f) None of the above.

Question 14

Your answer is INCORRECT.

Determine whether or not the vector functions are linearly dependent.

$$\mathbf{u} = \begin{bmatrix} 4t - 2 \\ -t \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -t + 2 \\ 4t \end{bmatrix}$$

a) Linearly dependent.

b) Linearly independent.

c) Cannot be determined.

Question 15

Your answer is INCORRECT.

Determine whether or not the vector functions are linearly dependent.

$$\mathbf{u} = \begin{bmatrix} -2\cos(t) \\ 3\sin(t) \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -2\sin(t) \\ 3\cos(t) \end{bmatrix}$$

a) Linearly dependent.

b) ○ Linearly independent.

c) ○ Cannot be determined.

Question 16

Your answer is INCORRECT.

Determine whether or not the vector functions are linearly dependent.

$$\mathbf{u} = \begin{bmatrix} 3-t \\ t \\ -3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} t \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -4+t \\ t+4 \\ -4 \end{bmatrix}$$

a) ◉ Linearly dependent.

b) ○ Linearly independent.

c) ○ Cannot be determined.

Question 17

Your answer is CORRECT.

Determine whether or not the vector functions are linearly dependent.

$$\mathbf{u} = \begin{bmatrix} 2\cos(t) \\ 2\sin(t) \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 2\cos(t) \\ 0 \\ 2\sin(t) \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 0 \\ 2\cos(t) \\ 2\sin(t) \end{bmatrix}$$

a) ◉ Linearly independent.

b) ○ Linearly dependent.

c) ○ Cannot be determined.

Question 18

Your answer is INCORRECT.

Given the linear differential system **x** ' = A**x** with

$$A = \begin{bmatrix} -5 & -2 \\ -7 & 0 \end{bmatrix}$$

Determine if **u**, **v** form a fundamental solution set. If so, give the general solution to the system.

$$u = \begin{bmatrix} e^{-7t} \\ e^{-7t} \end{bmatrix}, \quad v = \begin{bmatrix} 2e^{2t} \\ -7e^{2t} \end{bmatrix}$$

a) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = C \begin{bmatrix} e^t \\ e^t \end{bmatrix} + C \begin{bmatrix} e^{2t} \\ -e^{2t} \end{bmatrix}$

b) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = C_1 \begin{bmatrix} e^{-7t} \\ e^{-7t} \end{bmatrix} + C_2 \begin{bmatrix} e^{2t} \\ e^{2t} \end{bmatrix}$

c) *Not a fundamental solution set.*

d) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = C_1 \begin{bmatrix} e^{-7t} \\ e^{-7t} \end{bmatrix} + C_2 \begin{bmatrix} 2e^{2t} \\ -7e^{2t} \end{bmatrix}$

e) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e^{-7t} \\ e^{-7t} \end{bmatrix} + \begin{bmatrix} 2e^{2t} \\ -7e^{2t} \end{bmatrix}$

f) None of the above.

Question 19

Your answer is CORRECT.

Let X be the matrix function

$$X(t) = \begin{bmatrix} \cos(2t) & \sin(2t) \\ \sin(2t) & -\cos(2t) \end{bmatrix}$$

Determine if X is a fundamental matrix for system $\mathbf{x}' = A\mathbf{x}$ with

$$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

If so, find the solution of the system that satisfies

$$\mathbf{x}(0) = \begin{bmatrix} -3 \\ -5 \end{bmatrix}$$

a) *Not a fundamental solution set.*

b) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -3 \begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix} - 5 \begin{bmatrix} \sin(2t) \\ -\cos(2t) \end{bmatrix}$

c) ○ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix} + \begin{bmatrix} \sin(2t) \\ -\cos(2t) \end{bmatrix}$

d) ○ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -5\begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix} - 3\begin{bmatrix} \sin(2t) \\ -\cos(2t) \end{bmatrix}$

e) ◉ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -3\begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix} + 5\begin{bmatrix} \sin(2t) \\ -\cos(2t) \end{bmatrix}$

f) ○ None of the above.

Question 20

Your answer is INCORRECT.

Let X be the matrix function

$$X(t) = \begin{bmatrix} 0 & 4te^{-t} & e^{-t} \\ 1 & e^{-t} & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Determine if X is a fundamental matrix for system $\mathbf{x}' = A\mathbf{x}$ with

$$A = \begin{bmatrix} -1 & 4 & -4 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

If so, find the solution of the system that satisfies

$$\mathbf{x}(0) = \begin{bmatrix} -3 \\ -4 \\ -4 \end{bmatrix}$$

a) ○ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = -3\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - 4\begin{bmatrix} e^{-t} \\ 0 \\ 0 \end{bmatrix}$

b) ○ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = -4\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - 3\begin{bmatrix} e^{-t} \\ 0 \\ 0 \end{bmatrix}$

c) ◉ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = -4 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - 8 \begin{bmatrix} 4te^{-t} \\ e^{-t} \\ 0 \end{bmatrix} - 4 \begin{bmatrix} e^{-t} \\ 0 \\ 0 \end{bmatrix}$

d) ○ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = -3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - 4 \begin{bmatrix} 4te^{-t} \\ e^{-t} \\ 0 \end{bmatrix} - 3 \begin{bmatrix} e^{-t} \\ 0 \\ 0 \end{bmatrix}$

e) ○ *Not a fundamental solution set.*

f) ○ None of the above.