MATH 132 **EXAM III** FALL 2014

This exam has 18 questions. Part I will have 16 multiple choice questions, 5 points each. Part II will have 2 hand graded questions, 10 points each. Please check to see that your exam is complete. If you do not have a **pencil** to mark your card, please borow one from your proctor. **One page with formulas is attached to the end of this booklet.** Write your **ID NUMBER** (not your SS number) on the six blank lines on the top of your answer card, using one blank for each digit. After being sure of which answer you think is right, **shade in the corresponding box on your card.**

1) Use $\int \frac{du}{\sqrt{a^2+u^2}} = sinh^{-1}(\frac{u}{a}) + C$, for $a > 0$, $sinh^{-1}(x) = ln(x + \sqrt{x^2+1})$, to solve the integral $\int_0^{\sqrt{3}} \frac{1}{\sqrt{1+x^2}}\, dx$.

$A)\, ln(\sqrt{3}+1)$ $B)\, ln(\sqrt{3}+2)$ $C)\, ln(\sqrt{3}+4)$ $D)\, ln(\sqrt{3}+\sqrt{2})$ $E)\, ln(2\sqrt{3})$
$F)\, ln(3\sqrt{3})$ $G)\, ln(4)$ $H)\, ln(5)$ $I)\, ln(7)$ $J)\, ln(5\sqrt{3})$

2) Find the **limit** of the **sequence** $\left\{ (\frac{n+1}{2n})\,(1-\frac{1}{n}) \right\}_{n=1}^{\infty}$, if it converges .

A) 0 B) 1 C) $\frac{1}{2}$ $D)\, \frac{3}{2}$ $E)\, \frac{1}{3}$ $F)\, 2$ $G)\, \frac{2}{3}$ $H)\, \frac{4}{3}$ $I)\, \frac{1}{4}$ $J)$ diverges

3) Find the sum of the series $\{\frac{4}{9} + \frac{4}{27} + \frac{4}{81} + \frac{4}{243} + \dots\}$ if it converges.

A) $\frac{5}{9}$ B) $\frac{7}{93}$ $C)$ $\frac{10}{9}$ $D)$ $\frac{3}{4}$ $E)$ $\frac{5}{4}$ $F)$ $\frac{7}{4}$ $G)$ $\frac{2}{3}$ $H)$ $\frac{4}{3}$ $I)$ $\frac{5}{3}$ $J)$ *diverges*

4) Which of the following three series is/are **convergent** ?

I) $\sum_{n=1}^{\infty} \frac{n+2}{n^{3/2}}$ $II)$ $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^2}$ $III)$ $\sum_{n=1}^{\infty} \frac{n}{\sqrt{1+n^2}}$

A) I B) II C) III D) $I \& II$ E) $I \& III$ F) $II \& III$ G) I, II & III H) all diverge

5) Which of the following three series is/are **convergent** ?

I) $\sum_{n=2}^{\infty} \frac{1}{n\, ln(n)}$ II) $\sum_{n=2}^{\infty} \frac{ln(n)}{n}$ III) $\sum_{n=1}^{\infty} \frac{sin^2(n)}{n\sqrt{n}}$

A) I B) II C) III D) $I \,\&\, II$ E) $I \,\&\, III$ F) $II \,\&\, III$ G) I, II & III H) all diverge

6) If $S = \sum_{n=1}^{\infty} \frac{1}{n^3}$ and $S_{10} = \sum_{n=1}^{10} \frac{1}{n^3}$ ($10^{\underline{th}}$ partial sum) then using the **remainder estimate** of the **integral test,** we know that $R_{10} = (S - S_{10})$ is $\leq$:

A) 0.01 B) 0.001 C) 0.002 D) 0.03 E) 0.004 F) 0.05 G) 0.005 H) 0.06 I) 0.007

7) For which of the following 3 series will the **ratio test** be inconclusive . That means that the ratio test will not tell us if the series is convergent or divergent.

I) $\sum_{n=1}^{\infty} \frac{1}{n^3}$ $\quad$ $II)\ \sum_{n=1}^{\infty} \frac{2^n}{n^2}$ $\quad$ III) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$.

A) I B) II C) III D) I & II E) I & III F) II & III G) All H) None

8) Find the sum of the series $\sum_{n=1}^{\infty} \left(\frac{3}{n+1} - \frac{3}{n+2} \right)$

($Hint$: Find a formula for S_n).

A) 3 B) $\frac{1}{3}$ $C)\ \frac{2}{3}$ $D)\ \frac{3}{2}$ $E)\ 6$ $F)\ \frac{1}{6}$ $G)\ \frac{1}{2}$ $H)\ 2$ $I)\ \frac{5}{3}$ $J)$ diverges

9) Which of the following **alternating series** is/are **absolutely convergent** ?

I) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+5}$ $\quad$ $II)$ $\sum_{n=1}^{\infty}$ (-1)$^{n+1}$ $\frac{sin^2(n)}{n^2}$ $\quad$ $III)$$\sum_{n=1}^{\infty}$ (-1)$^{n+1}$ $\frac{n^3+1}{n^4+1}$

A) I B) II C) III D) I & II E) I & III F) II & III G) All H) None

10) Using geometric series rewrite the decimal **0.A777777 . . .** , as a fraction.
(**A** is some integer between 0 and 9.)

A) $\frac{9A+7}{90}$ B) $\frac{7A+9}{99}$ C) $\frac{6A+8}{90}$ D) $\frac{8A+6}{99}$ E) $\frac{7A+4}{90}$ F) $\frac{4A+7}{99}$ G) $\frac{4A+6}{90}$ H) $\frac{6A+4}{99}$

11) Use the root test to determine which of the following three series is/are **convergent**.

I) $\sum_{n=1}^{\infty} \left(\frac{n}{2n+3} \right)^n$ $\quad$ $II)$ $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$ $\quad$ $III)$ $\sum_{n=1}^{\infty} \frac{8}{(3+\frac{1}{n})^{2n}}$

A) I B) II C) III D) $I \,\&\, II$ E) $I \,\&\, III$ F) $II \,\&\, III$ G) I, II & III H) all diverge

12) Find the sum of the infinite series $\sum_{n=1}^{\infty} \frac{2^n+3^n}{4^{n-1}}$.

$A)$ 5 $B)$ 6 $C)$ 7 $D)$ 8 E) 10 $F)$ 12 $G)$ 14 $H)$ 16 $I)$ 18 $J)$ *series diverges*

13) For the series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{2n^2+4}$, find the smallest n for which the partial sum S_n will be an approximation of the sum of the series with an error less than 0.001 ?

$A)$ 8 $B)$ 12 $C)$ 16 $D)$ 18 $E)$ 22 $F)$ 24 $G)$ 28 $H)$ 32 I) 36 J) 40

14) Find the complete <u>interval of convergence</u> (either absolute or conditional) of the power series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^3 3^n}$.

$A)\ -1 < x < 3$ $B)\ -1 < x \leq 3$ $C)\ -1 \leq x < 3$ $D)\ -1 \leq x \leq 3$ $E)\ -2 < x < 4$
$F)\ -2 < x \leq 4$ $G)\ -2 \leq x < 4$ $H)\ -2 \leq x \leq 4$ $I)\ -\infty < x < \infty$ $J)\ \{1\}$

15) Find the Taylor polynomial of order 2 for $tan(x)$ at $a = \frac{\pi}{4}$.

$A)\, 1 + (x - \frac{\pi}{4}) + (x - \frac{\pi}{4})^2$ $B)\, 1 + (x - \frac{\pi}{4}) + 2(x - \frac{\pi}{4})^2$
$C) 1 + 2(x - \frac{\pi}{4}) + 2(x - \frac{\pi}{4})^2$ $D)\ 1 + 2(x - \frac{\pi}{4}) + 4(x - \frac{\pi}{4})^2$
$E)\ 2 + 2(x - \frac{\pi}{4}) + 2(x - \frac{\pi}{4})^2$ $F)\ 2 + 2(x - \frac{\pi}{4}) + 4(x - \frac{\pi}{4})^2$
$H)\, 2 + 4(x - \frac{\pi}{4}) + 4(x - \frac{\pi}{4})^2$ $I)\ 2 + 4(x - \frac{\pi}{4}) + 8(x - \frac{\pi}{4})^2$

16) If $\sum\limits_{n=0}^{4} c_n x^n$ is the Taylor polynomial of order 4 generated by $f(x) = \cos(\frac{x}{2})$ at $a = 0$, then find c_4, the constant in front of x^4.

$A) \frac{1}{242}$ $B) -\frac{1}{242}$ $C) \frac{1}{283}$ $D) -\frac{1}{283}$ $E) \frac{1}{324}$ $F) -\frac{1}{324}$ $G) \frac{1}{384}$ $H) -\frac{1}{384}$
$I) \frac{1}{412}$ $J) -\frac{1}{412}$

PART II: CLEARLY WRITE YOUR SOLUTION AND HOW YOU GOT IT.
Your section letter appears on the front page of the exam

Name : ______________________ **ID # :** _____________ **Subsec :** _______

17) The series $3 + \frac{6}{5} + \frac{12}{25} + \frac{24}{125} + \frac{48}{625} + \ldots\ldots = \sum_{n=0}^{\infty} a\, r^n$, is a geometric series .

a) What are the numbers a and r, and find what the series **converges to** .

b) Use the **integral test** to show that the above series converges .

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18) a) Use the **ratio test** to determine if the series $\sum_{n=1}^{\infty} \frac{2^n}{n \cdot 3^n}$, **converges** or **diverges**.
If the test will not work in this case, say that it is **inconclusive**.

b) Use the **comparison** or **limit comparison test** to see if the series $\sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2}$ **converges** or **diverges** . Also, **explain why** the series you are comparing it to converges or diverges.