

Calculus II - Fall 2013

Midterm Exam II, November 12, 2013

In the following problems you are required to show all your work and provide the necessary explanations everywhere to get full credit.

1. Find the area between the graphs of $y = \cos x$ and $y = \sin 2x$ for $0 \leq x \leq \pi/2$.

2. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, $x = 1$, and $y = 0$

(a) about the x -axis.

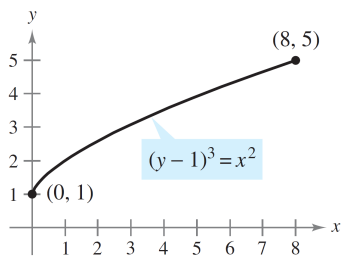
(b) about the line $x = 1$.

(c) about the y -axis.

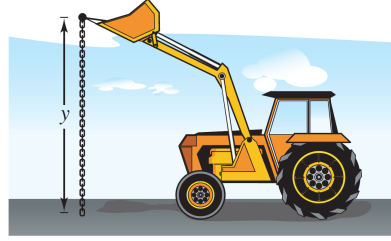
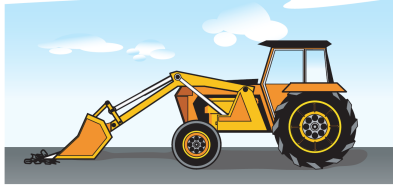
(d) about the line $y = -1$.

3. Find the volume of the solid obtained by rotating the region bounded by $y = x^3 + x + 1$, $x = 1$, and $y = 1$ about the line $x = 2$.

4. Find the arc length of the graph of $(y - 1)^3 = x^2$ on the interval $[0, 8]$ as shown in the Figure below.

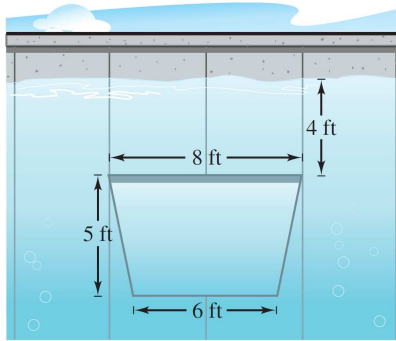


5. A 20-foot chain weighing 5 pounds per foot is lying coiled on the ground. How much work is required to raise one end of the chain to a height of 20 feet so that it is fully extended?



6. A spherical tank of radius 8 feet is half full of oil that weighs 50 pounds per cubic foot. Find the work required to pump oil out through a hole in the top of the tank.

7. A vertical gate in a dam has the shape of an isosceles trapezoid 8 feet across the top and 6 feet across the bottom, with a height of 5 feet, as shown in the Figure below. What is the fluid force on the gate when the top of the gate is 4 feet below the surface of the water?



8. Find the center of mass of the lamina of uniform density ρ bounded by the graph of $f(x) = 4 - x^2$ and the x -axis.

9. Given the initial condition $y(0) = 1$, find the particular solution of the equation

$$xydx + e^{-x^2}(y^2 - 1)dy = 0$$

10. Match the differential equation with its slope field. **Explain!**

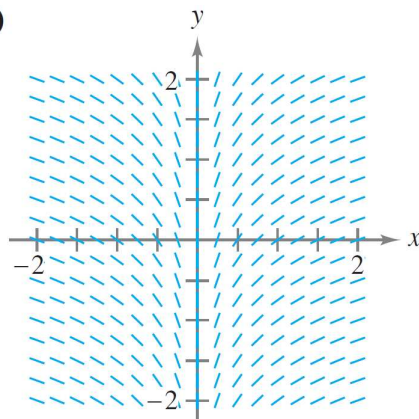
(I) $\frac{dy}{dx} = \cos 2x$

(II) $\frac{dy}{dx} = \frac{1}{2} \sin x$

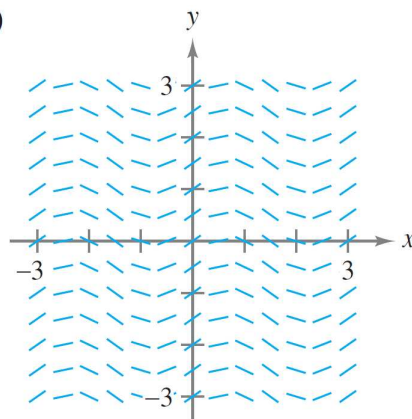
(III) $\frac{dy}{dx} = e^{-2x}$

(IV) $\frac{dy}{dx} = \frac{1}{x}$

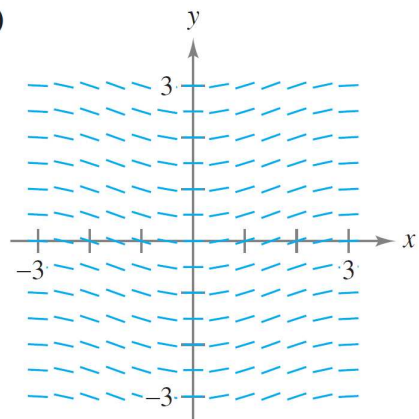
(a)



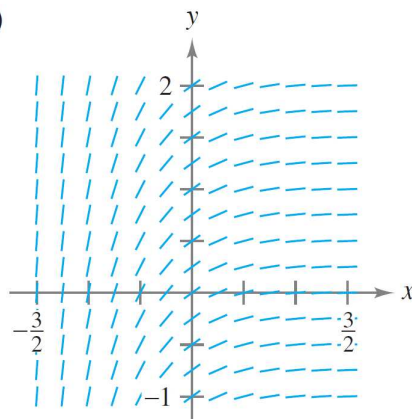
(b)



(c)



(d)



11. Find a sequence $\{a_n\}$ whose first five terms are

$$\frac{1}{7}, \frac{3}{9}, \frac{7}{11}, \frac{15}{13}, \frac{31}{15}, \dots$$

Then determine whether the particular sequence you have chosen converges or diverges.

12. Prove that the sequence $a_n = \frac{2n}{1+n}$ is monotonic.