Math 2413 Instructor: Adnan Said
Chapter 2: Sections 2.1, 2.2, 2.3, 2.4, 2.5, and 2.6

2.1 The Derivative and the Tangent Line Problem

Tangent Lines to the Graph of a function

If ((c, f(c)) is a point on a graph of f, and (c + △x, f(c + △x)) is a second point on the graph of f, then the slope of the secant line through the two points is:

$$m = \frac{f(c + \triangle x) - f(c)}{\triangle x}$$

We can obtain more and more approximations of the slope of the tangent line by choosing points closer and closer to the point of tangency.
The slope of the tangent line to the graph of f at the point (c , f(c)) is:

$$m = \lim_{\triangle x \to 0} \frac{f(c + \triangle x) - f(c)}{\triangle x}$$

Definition of the derivative of a function

The derivative of a function f(x) gives the slope of the tangent line to the graph of f at ((x, f(x)).Therefore, the derivative of f at x is given by

$$f'(x) = \lim_{\triangle x \to 0} \frac{f(x + \triangle x) - f(x)}{\triangle x}$$ **provided the limit exists.**

Find the derivative by the limit process.

1) $f(x) = x^2 - 4x.$ 1) ______________

$$f'(x) = \lim_{\triangle x \to 0} \frac{f(x + \triangle x) - f(x)}{\triangle x}$$

$$= \lim_{\triangle x \to 0} \frac{\left[(x + \triangle x)^2 - 4(x + \triangle x)\right] - \left[x^2 - 4x\right]}{\triangle x}$$

$$= \lim_{\triangle x \to 0} \frac{x^2 + 2x(\triangle x) + (\triangle x)^2 - 4x - 4\triangle x - x^2 + 4x}{\triangle x}$$

$$= \lim_{\triangle x \to 0} \frac{2x(\triangle x) + (\triangle x)^2 - 4\triangle x}{\triangle x}$$

$$= \lim_{\triangle x \to 0} \frac{\triangle x(2x + \triangle x - 4)}{\triangle x}$$

$$= \lim_{\triangle x \to 0} (2x + \triangle x - 4) = 2x - 4$$

Find the equation of the tangent line to the graph of f(x) at the point where x = 1.

2) $f(x) = x^2 - 4x.$ 2) ______________

From problem # 1, $f'(x) = 2x - 4$.

The slope of the tangent line at x = 1 is:

$m = f'(1) = 2(1) - 4 = -2.$

The equation of the tangent line is: $y - y_1 = m(x - x_1)$

$\rightarrow y + 3 = -2(x - 1) \rightarrow y + 3 = -2x + 2 \rightarrow 2x + y = -1$

Notations for derivatives

The most common notations are: $f'(x)$, $\frac{dy}{dx}$, y', $\frac{d}{dx}[f(x)]$, $D_x[y]$.

The alternative form of derivative

The derivative of f at c is: $f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ provided the limit exists.

Use the alternative form of derivative to find the derivative at x = c (if it exists).

3) $f(x) = x^3 + 6x$, $c = 2.$ 3) ______________

$$f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} \frac{x^3 + 6x - 20}{x - 2} = \frac{0}{0}$$

2	1	0	6	−20
		2	4	20
	1	2	10	0

$$\frac{x^3 + 6x - 20}{x - 2} = \frac{(x - 2)(x^2 + 2x + 10)}{(x - 2)} = x^2 + 2x + 10$$

So, $$\lim_{x \to 2} \frac{x^3 + 6x - 20}{x - 2} = \lim_{x \to 2} (x^2 + 2x + 10) = 18$$

Existence of the Derivative

A function is differentiable at x if its derivative exists at x.

A function is not differentiable in three cases:

a) at a point where the function itself is not defined.

b) at "corners" or "sharp points".

c) at a point where the function has a vertical tangent.

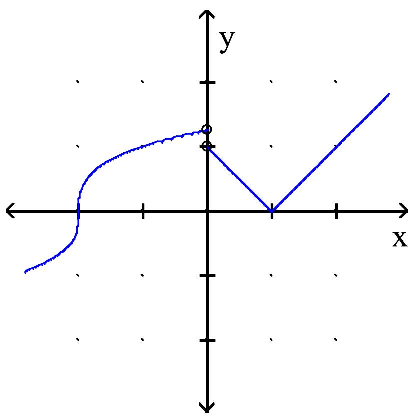
Relationship Between Continuity and Differentiability

a) If a function is differentiable at x = c, then it is continuous at x = c.

b) It is possible for a function to be continuous at x = c and not be differentiable at x = c.

Find all points where the function whose graph shown do not have derivatives.

4)



4) ______________

The function is not differentiable at $x = -2$, because the tangent line at $x = -2$ is vertical.

The function is not differentiable at $x = 0$, because at $x = 0$ the function is not defined.

The function is not differentiable at $x = 1$, because at $x = 1$ there is a sharp corner.

2.1 Exercises pg 103 **(25, 66, 77)** **(26, 68, 79)**

2.2 Basic Differentiation Rules

Basic Differentiation Rules

The Sum or Difference Rules: $\frac{d}{dx}[\,u(x) \pm v(x)\,] = u'(x) \pm v'(x)$

The Constant Multiple Rule: $\frac{d}{dx}[c\,u(x)\,] = c\,u'(x)$ The Constant Rule: $\frac{d}{dx}[c] = 0$

The Sine Function Rule: $\frac{d}{dx}[\sin x] = \cos x$ The Linear Term Rule: $\frac{d}{dx}[\,c\,x\,] = c$

The Cosine Function Rule: $\frac{d}{dx}[\cos x] = -\sin x$ The Power Rule: $\frac{d}{dx}[x^n] = nx^{n-1}$

Find the derivative.

5) $y = 8x^{-3} + 6x^4 - 2x + 7$ 5) ______

$y' = 8(-3)x^{-4} + 6(4)x^3 - 2 = -24x^{-4} + 24x^3 - 2$

6) $y = 3x^{6/5} - 4x^3 - 10x$ 6) ______

$y' = 3(\frac{6}{5})x^{1/5} - 4(3)x^2 - 10 = \frac{18}{5}x^{1/5} - 12x^2 - 10$

7) $y = \pi \sin x - \frac{5\cos x}{3} = \pi \sin x - \frac{5}{3}\cos x$ 7) ______

$y' = \pi \cos x - \frac{5}{3}(-\sin x) = \pi \cos x + \frac{5}{3}\sin x$

Find the following.

8) $f'(4)$ if $f(x) = \frac{6x - 4}{\sqrt{x}}$ 8) ______

$f(x) = \frac{6x - 4}{x^{1/2}} = (6x - 4)x^{-1/2} = 6x^{1/2} - 4x^{-1/2}$

$f'(x) = 6(\frac{1}{2})x^{-1/2} - 4(-\frac{1}{2})x^{-3/2}$

$= 3x^{-1/2} + 2x^{-3/2} = \frac{3}{x^{1/2}} + \frac{2}{x^{3/2}}$

$f'(4) = \frac{3}{4^{1/2}} + \frac{2}{4^{3/2}} = \frac{3}{2} + \frac{2}{8} = \frac{3}{2} + \frac{1}{4} = \frac{7}{4}$

9) **$f'(2)$ if $f(x) = \dfrac{2x^7 - 4x^2}{x^4}$** 9) ____________

$f(x) = (2x^7 - 4x^2)x^{-4} = 2x^3 - 4x^{-2}$

$f'(x) = 2(3)x^2 - 4(-2)x^{-3} = 6x^2 + 8x^{-3}$

$f'(2) = 6(2)^2 + 8(2)^{-3} = 6(4) + 8(\frac{1}{8}) = 24 + 1 = 25$

Find the equation of the tangent line to the curve when x has the given value.

10) **$f(x) = 3x^{4/3} - 2x^2$ at $x = 8$** 10) ____________

$f'(x) = 3(\frac{4}{3})x^{1/3} - 2(2)x = 4x^{1/3} - 4x$

The slope of the tangent line at x = 8 is

$m = f'(8) = 4(8)^{1/3} - 4(8) = -24$.

$f(8) = 3(8)^{4/3} - 2(8)^2 = -80$.

The equation of the tangent line is: $y - y_1 = m(x - x_1)$

$\rightarrow y + 80 = -24(x - 8) \rightarrow y + 80 = -24x + 192 \rightarrow 24x + y = 112$

Find all values of x (if any) where the tangent line to the graph of the function is horizontal.

11) **$y = \sqrt{3}x + 2\cos x \;;\; 0 \le x < 2\pi$** 11) ____________

$y' = \sqrt{3} + 2(-\sin x) = \sqrt{3} - 2\sin x$

The tangent line to the graph of the function is horizontal,

$f'(x) = 0 \rightarrow \sqrt{3} - 2\sin x = 0 \rightarrow \sin x = \dfrac{\sqrt{3}}{2} \rightarrow x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

2.2 Exercises pg 114

(21, 35, 44, 51, 53, 57, 58) (23, 37, 45, 52, 56, 61)

2.3 Product and Quotient Rules and Higher–Order Derivatives

The Product Rule: $\frac{d}{dx}[u(x) \cdot v(x)] = u'(x) \cdot v(x) + v'(x) \cdot u(x)$

Find the derivative of the function.

12) $f(x) = (6x^3 - x^2)(4x - 3)$ 12) ____________

$u(x) = 6x^3 - x^2 \rightarrow u'(x) = 18x^2 - 2x \; ; \; v(x) = 4x - 3 \rightarrow v'(x) = 4$

$f'(x) = u'(x) \cdot v(x) + v'(x) \cdot u(x)$

$= (18x^2 - 2x)(4x - 3) + 4(6x^3 - x^2)$

$= 72x^3 - 54x^2 - 8x^2 + 6x + 24x^3 - 4x^2 = 96x^3 - 66x^2 + 6x$

13) $f(x) = (\sqrt{x} - 3)(x^2 + 4)$ 13) ____________

$u(x) = x^{1/2} - 3 \rightarrow u'(x) = \frac{1}{2}x^{-1/2} \quad ; \quad v(x) = x^2 + 4 \rightarrow v'(x) = 2x$

$f'(x) = u'(x) \cdot v(x) + v'(x) \cdot u(x)$

$= \frac{1}{2}x^{-1/2}(x^2 + 4) + 2x(x^{1/2} - 3)$

$= \frac{1}{2}x^{3/2} + 2x^{-1/2} + 2x^{3/2} - 6x = \frac{5}{2}x^{3/2} + 2x^{-1/2} - 6x$

The Quotient Rule: $\frac{d}{dx}\left[\frac{u(x)}{v(x)}\right] = \frac{u'(x) \cdot v(x) - v'(x) \cdot u(x)}{[v(x)]^2}$

Find the derivative of the function.

14) $f(x) = \frac{4x - 3}{3x^2 - 2}$ 14) ____________

$u(x) = 4x - 3 \rightarrow u'(x) = 4 \quad ; \quad v(x) = 3x^2 - 2 \rightarrow v'(x) = 6x$

$f'(x) = \frac{4(3x^2 - 2) - 6x(4x - 3)}{(3x^2 - 2)^2} = \frac{12x^2 - 8 - 24x^2 + 18x}{(3x^2 - 2)^2}$

$= \frac{-12x^2 + 18x - 8}{(3x^2 - 2)^2}$

15) $y = \dfrac{\sqrt{x}}{2x^2 - 4} = \dfrac{x^{1/2}}{2x^2 - 4}$ 15) ____________

$u(x) = x^{1/2} \rightarrow u'(x) = \dfrac{1}{2}x^{-1/2}$; $v(x) = 2x^2 - 4 \rightarrow v'(x) = 4x$

$$f'(x) = \frac{\frac{1}{2}x^{-1/2}(2x^2 - 4) - 4x(x^{1/2})}{(2x^2 - 4)^2} = \frac{x^{3/2} - 2x^{-1/2} - 4x^{3/2}}{(2x^2 - 4)^2}$$

$$= \frac{-3x^{3/2} - 2x^{-1/2}}{(2x^2 - 4)^2}$$

The Tangent and Cotangent Functions Rules

$\dfrac{d}{dx}[\tan x] = \sec^2 x$; $\dfrac{d}{dx}[\cot x] = -\csc^2 x$

The Secant and Cosecant Functions Rules

$\dfrac{d}{dx}[\sec x] = \sec x \tan x$; $\dfrac{d}{dx}[\csc x] = -\csc x \cot x$

Find the derivative of the function.

16) $f(x) = 3x^2 \sin x$ 16) ____________

$u(x) = 3x^2 \rightarrow u'(x) = 6x$; $v(x) = \sin x \rightarrow v'(x) = \cos x$

$f'(x) = u'(x) \cdot v(x) + v'(x) \cdot u(x)$

$= (6x)(\sin x) + (\cos x)(3x^2) = 6x \sin x + 3x^2 \cos x$

17) $y = \dfrac{\sin x}{\cos x - 1}$ 17) ____________

$u(x) = \sin x \rightarrow u'(x) = \cos x$; $v(x) = \cos x - 1 \rightarrow v'(x) = -\sin x$

$$f'(x) = \frac{\cos x(\cos x - 1) + \sin x(\sin x)}{(\cos x - 1)^2} = \frac{\cos^2 x - \cos x + \sin^2 x}{(\cos x - 1)^2}$$

$$= \frac{1 - \cos x}{(\cos x - 1)^2} = \frac{(1 - \cos x)}{(\cos x - 1)(\cos x - 1)} = \frac{-1}{(\cos x - 1)} \text{ or } \frac{1}{1 - \cos x}$$

Higher – Order Derivatives

$f''(x)$ is the second derivative of the function also written as $\frac{d^2y}{dx^2}$.

$f'''(x)$ is the third derivative of the function also written as $\frac{d^3y}{dx^3}$.

$f^{(n)}(x)$ is the nth derivative of the function also written as $\frac{d^ny}{dx^n}$.

Find the third derivative of the function.

18) $f(x) = 2x^4 - 4x^3 - 3x^2 - 7x + 2.$ 18) ____________

$f'(x) = 8x^3 - 12x^2 - 6x - 7$

$f''(x) = 24x^2 - 24x - 6$; $f'''(x) = 48x - 24$

19) $f(x) = \frac{-16}{\sqrt{x}} + x^3 - 4x + 9 = -16x^{-1/2} + x^3 - 4x + 9$ 19) ____________

$f'(x) = 8x^{-3/2} + 3x^2 - 4$

$f''(x) = -12x^{-5/2} + 6x$; $f'''(x) = 30x^{-7/2} + 6$

20) $f(x) = 3 \sin x + 2 \cos x$ 20) ____________

$f'(x) = 3 \cos x - 2 \sin x$

$f''(x) = -3 \sin x - 2 \cos x$; $f'''(x) = -3 \cos x + 2 \sin x$

2.3 Exercises pg 125
(1, 8, 15, 17, 41, 50, 63, 73, 95, 100)
(5, 11, 33, 39, 48, 51, 65, 75, 97, 101)

2.4 The Chain Rule

The Chain Rule

If y is a function of u, and u is a function of x, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

A special case of the chain rule is the generalized power rule.

Generalized Power Rule

Let u be a function of x, and let $y = u^n$. Then $y' = n \cdot u^{n-1} \cdot u'$

Find the derivative of the function.

21) $y = (2x^2 + 3x)^7$ 21) ____________

$y' = 7(2x^2 + 3x)^6(4x + 3)$

22) $y = -6\sqrt{5x + 6} = -6(5x + 6)^{1/2}$ 22) ____________

$y' = -6\left(\frac{1}{2}\right)(5x + 6)^{-1/2}(5)$

$= -15(5x + 6)^{-1/2} = \frac{-15}{(5x + 6)^{1/2}} = \frac{-15}{\sqrt{(5x + 6}}$

23) $f(x) = \frac{-5}{(4x - 3)^4} = -5(4x - 3)^{-4}$ 23) ____________

$f'(x) = (-5)(-4)(4x - 3)^{-5}(4) = 80(4x - 3)^{-5} = \frac{80}{(4x - 3)^5}$

Solve the problem.

24) Find all points on the graph of $f(x) = \sqrt[3]{(x^3 - 8)^2}$ for which $f'(x) = 0$ and those for which $f'(x)$ does not exist. 24) ____________

$f(x) = (x^3 - 8)^{2/3}$

$f'(x) = \left(\frac{2}{3}\right)(x^3 - 8)^{-1/3}(3x^2) = \frac{2x^2}{(x^3 - 8)^{1/3}}$ or $\frac{2x^2}{\sqrt[3]{x^3 - 8}}$

$f'(x) = 0$ when $x = 0$; $f'(x)$ does not exist when $x = 2$

The Chain Rule versions of the derivatives of the six trigonometric functions are:

The Sine and Cosine Functions Rules

$\frac{d}{dx}[\sin u] = (\cos u)\, u'$; $\frac{d}{dx}[\cos u] = -(\sin u)\, u'$

The Tangent and Cotangent Functions Rules

$\frac{d}{dx}[\tan u] = (\sec^2 u)\, u'$; $\frac{d}{dx}[\cot u] = -(\csc^2 u)\, u'$

The Secant and Cosecant Functions Rules

$\frac{d}{dx}[\sec u] = (\sec u \tan u)\, u'$; $\frac{d}{dx}[\csc u] = -(\csc u \cot u)\, u'$

Find the derivative of the function.

25) $f(x) = \tan(4x) \rightarrow f'(x) = 4(\sec^2(4x))$ 25) ____________

26) $f(x) = \cos(3x)^2 = \cos(9x^2) \rightarrow f'(x) = -18x\sin(9x^2)$ 26) ____________

27) $f(x) = \sin^3 4x = (\sin 4x)^3$ 27) ____________

$f'(x) = 3(\sin 4x)^2 (\cos 4x)(4) = 12\sin^2 4x \cos 4x$

Solve the problem.

28) Let $f(x) = 2\sin x + \cos 2x$. 28) ____________

Find the tangent to the graph at the point $(\pi, 1)$.

$f'(x) = 2\cos x - 2\sin 2x$

The slope of the tangent line at $x = \pi$ is $m = f'(\pi) = -2$

The equation of the tangent line is:

$y - y_1 = m(x - x_1) \rightarrow y - 1 = -2(x - \pi) \rightarrow y = -2x + 2\pi + 1$

29) Let $f(x) = 2\sin x + \cos 2x$. 29) ____________

Determine all the values of x in the interval $(0, 2\pi)$ at which the graph of f has a horizontal tangent.

$f'(x) = 2\cos x - 2\sin 2x = 2\cos x - 2(2\sin x\cos x)$

$= 2\cos x - 4\sin x\cos x = 2\cos x(1 - 2\sin x)$

The graph of f has a horizontal tangent when $f'(x) = 0$

$2\cos x(1 - 2\sin x) = 0 \rightarrow 2\cos x = 0$ or $1 - 2\sin x = 0$

$\rightarrow \cos x = 0$ or $\sin x = \frac{1}{2} \rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

2.4 Exercises pg 136

(9, 20, 49, 56, 65, 76, 85, 91) (13, 22, 55, 57, 68, 79, 83, 89, 92)

2.5 Implicit differentiation

Implicit and Explicit Functions

A function that is implied by an equation is said to be written in implicit form. To differentiate a function that is written in implicit form, change the equation to explicit form, and then differentiate. Implicit differentiation is used when it is very difficult to express y as a function of x explicitly. When differentiating terms involving x alone, you can differentiate as usual. However, when differentiating terms involving y, you must apply the chain rule, because y is a function of x.

Guidelines for Implicit Differentiation

1. Differentiate both sides of the equation with respect to x.
2. Collect all terms involving y' on the left side of the equation and move all other terms to the right side of the equation.
3. Factor y' out of the left side of the equation, and solve for y'.

Find y'.

30) $y^2 - 5y - x^2 = -4$ 30) ____________

Solution:

$2yy' - 5y' - 2x = 0 \rightarrow 2yy' - 5y' = 2x$

$\rightarrow y'(2y - 5) = 2x \rightarrow y' = \dfrac{2x}{2y - 5}$

31) $xy^2 - 2y = x$ 31) ____________

Solution:

$u = x \rightarrow u' = 1\ ;\quad v = y^2 \rightarrow v' = 2yy'$

$1(y^2) + 2yy'(x) - 2y' = 1 \rightarrow y^2 + 2yy'x - 2y' = 1$

$\rightarrow 2yy'x - 2y' = 1 - y^2 \rightarrow y'(2yx - 2) = 1 - y^2 \rightarrow y' = \dfrac{1 - y^2}{2yx - 2}$

32) $y^4 + x^3 = y^2 + 10x$ 32) ____________

Solution:

$4y^3y' + 3x^2 = 2yy' + 10 \rightarrow 4y^3y' - 2yy' = -3x^2 + 10$

$\rightarrow y'(4y^3 - 2y) = -3x^2 + 10 \rightarrow y' = \dfrac{-3x^2 + 10}{4y^3 - 2y}$

33) $4x^2y - \pi \cos y = 5\pi$ 33) ____________

Solution:

$u = 4x^2 \rightarrow u' = 8x \quad ; \quad v = y \rightarrow v' = y'$

$(8x)y + y'(4x^2) - \pi\,(-\sin y)\, y' = 0 \rightarrow 8xy + 4y'x^2 + \pi \sin y\, y' = 0$

$\rightarrow 4y'x^2 + \pi \sin y\, y' = -8xy \rightarrow y'(4x^2 + \pi \sin y) = -8xy$

$$\rightarrow y' = \frac{-8xy}{4x^2 + \pi \sin y}$$

Find the equation of the tangent line at the indicated point on the given curve.

34) $y^4 + x^3 = y^2 + 10x$, tangent at $(0, 1)$ 34) ____________

Solution: $y' = \dfrac{-3x^2 + 10}{4y^3 - 2y}$

The slope of the tangent line at (0, 1) is $\dfrac{-3(0)^2 + 10}{4(1)^3 - 2(1)} = 5$

The equation of the tangent line at (0, 1) is:

$y - y_1 = m\,(x - x_1) \rightarrow y - 1 = 5(x - 0) \rightarrow y = 5x + 1$

35) $4x^2y - \pi \cos y = 5\pi$, tangent at $(1, \pi)$ 35) ____________

Solution: $y' = \dfrac{-8xy}{4x^2 + \pi \sin y}$

The slope of the tangent line at $(1, \pi)$ is:

$$m = \frac{-8(1)(\pi)}{4(1)^2 + \pi \sin(\pi)} = \frac{-8\pi}{4} = -2\pi$$

The equation of the tangent line at $(1, \pi)$:

$y - y_1 = m\,(x - x_1) \rightarrow y - \pi = -2\pi(x - 1)$

$\rightarrow y - \pi = -2\pi x + 2\pi \rightarrow y = -2\pi x + 3\pi$

2.5 Exercises pg 145 (5, 11, 26, 29) (7, 14, 28, 30)

2.6 Related Rates

Implicit differentiation is useful in finding the rates of change of two or more related variables that are changing with respect to time t.

Guidelines for Solving Related–Rate Problems

1. Identify all given quantities and quantities to be determined.
2. Write an equation involving the variables whose rates of change either are given or are to be determined.
3. Differentiate implicitly both sides of the equation with respect to time t.
4. Substitute into the resulting equation all known values for the variables and their rates of change. Solve for the required rate of change.

Solve the problem.

36) **A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 1.5 foot per second. When the radius is 4 feet, at what rate is the area A of the disturbed water changing?** 36) ____________

Solution:

1. Given rate: $\frac{dr}{dt} = 1.5$ Find: $\frac{dA}{dt}$ when $r = 4$
2. Equation: $A = \pi r^2$
3. Differentiate implicitly: $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$
4. Substitute all known values in the above equation, and solve for $\frac{dA}{dt}$.

$$\frac{dA}{dt} = 2\pi(4)(1.5) = 12\pi \text{ square feet per second}$$

When the radius is 4 feet, the area is changing at a rate of 12π square feet per second.

37) **Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet.** 37) ____________

Solution:

1. Given rate: $\frac{dV}{dt} = 4.5$ Find: $\frac{dr}{dt}$ when $r = 2$
2. Equation: $V = \frac{4}{3}\pi r^3$ (Volume of a sphere)
3. Differentiate implicitly: $\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2\frac{dr}{dt} \rightarrow \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$
4. Substitute all known values in the above equation, and solve for $\frac{dr}{dt}$.

   We get $4.5 = 16\pi\frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{4.5}{16\pi} \approx 0.09$ foot per minute.

When the radius is 2 feet, the volume is changing at a rate of 0.09 foot per minute.

2.6 Exercises pg 153 (3, 11, 15) (4, 16, 17)